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Experimental implementation of a detection scheme for entanglement in a spinor Bose gas

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Experimental implementation of a detection scheme for entanglement in a spinor Bose gas

The detection of entanglement in many-body quantum systems represents a major challenge for modern ultracold gas experiments. This thesis presents the experimental implementation of a newly developed scheme for the detection of entanglement using a joint measurement of two non-commuting observables. In our experiments we employ a spinor Bose-Einstein condensate of ^{87}Rb , which allows the investigation of spin dynamics involving few as well as multiple spatial modes. We introduce the relevant theoretical concepts and analyze the dynamics based on simulations in the truncated Wigner approximation. To reach a high level of stability in our experiment a newly developed arbitrary waveform generator setup is presented, enabling well controlled and flexible manipulations of the collective spin state. Further, we discuss the experimental implementation and the calibration of the readout scheme, that are necessary for the precise detection of entanglement. Additionally, we test the influence of external fluctuations simulating the experimental readout sequence. Finally, we present first experimental results showing entanglement in a controlled few mode setting and simultaneous squeezing of four spatial modes in a box-like trapping potential.

Experimentelle Implementierung eines Detektionsschemas für Quantenverschränkung in einem Spinor-Bose-Gas

Der Nachweis von Quantenverschränkung in Vielkörper-Quantensystemen stellt eine große Herausforderung für moderne ultrakalte Gasexperimente dar. In dieser Arbeit wird die experimentelle Implementierung eines neu entwickelten Schemas zum Nachweis von Quantenverschränkung vorgestellt, welches eine gleichzeitige Messung von zwei nicht kommutierenden Observablen erlaubt. Zur Untersuchung von Spin-Dynamik in wenig- und Vielmoden Situationen wird in unseren Experimenten ein Spinor Bose-Einstein-Kondensat bestehend aus ^{87}Rb verwendet. Zuerst werden die relevanten theoretischen Konzepte vorgestellt und die Dynamik basierend auf Simulationen in der truncated Wigner Näherung analysiert. Um in unserem Experiment ein hohes Maß an Stabilität zu erreichen, wird ein neu entwickelter Aufbau eines Arbiträr Funktionsgenerators vorgestellt, der eine gut kontrollierte und flexible Manipulation des atomaren Spinzustands ermöglicht. Des Weiteren werden das Ausleseschema und die experimentellen Details bezüglich der Implementierung und Kalibrierung diskutiert, die für die genaue Detektion von Quantenverschränkung erforderlich sind. Zusätzlich wird der Einfluss externer Fluktuationen unter Einbeziehung der experimentellen Sequenz mit Hilfe von Simulationen untersucht. Schließlich werden erste experimentelle Ergebnisse präsentiert, die zum einen die Verschränkung in einer kontrollierten Situation mit wenigen Moden und darüber hinaus das gleichzeitige Quetschen von vier Moden in einem kastenartigen Potential zeigen.

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1 Introduction

Entanglement as a quantum mechanics phenomenon has been extensively studied in theory [1] and experiments [2]. While the theory gives a clear definition of entanglement using the inseparability of entangled states [3] the experimental verification is a challenging task. Due to the fragility with respect to the environment [4] and the complex structure [5] of entanglement, the experimental detection demands a high accuracy.

First progress was made in few particle systems. Here, the control and detection on the single particle level allowed the investigation of entanglement in different experiments over the last years [6]. However, the extension to many-body systems has been a difficult task and has only been investigated recently [7, 8]. To adapt the definition to many-body systems consisting of indistinguishable particles, it has been shown, that entanglement between these particles is directly accessible via entanglement in-between distinguishable modes [9]. This sort of entanglement is solely transferred from the single particles to the distinguishable modes by local operations, which do not generate entanglement themselves. A platform to study these collective systems and questions related to entanglement are Bose-Einstein condensates. Due to their high controllability and their isolation from the environment they provide a suitable platform. Still, the experimental detection of entanglement in a collective state of indistinguishable particles in general [10, 11] as well as its verification between two spatially distinct regions [12, 13] remains a major challenge.

In our experiments we employ a quasi-one-dimensional spin-1 Bose-Einstein condensate (BEC) of Rubidium (^{87}Rb). Having a high control of the platform, we apply a recently employed simultaneous readout scheme of two non-commuting observables [14]. Paired with our spatially resolved absorption imaging, the setup provides the possibility to investigate entanglement in a continuous variable system.

To tackle the challenge of detecting entanglement, the experimental control has been improved over the years [15, 16, 17, 18, 19]. This foundation allows to apply a simultaneous readout scheme of the spin-1 observables S_x and Q_{yz} . Considering the scheme's proneness for external fluctuations we carefully calibrate the experimental system and the readout scheme. Employing spin-mixing dynamics we detect fluctuations below those of a coherent state for a squeezed state in a well controlled two mode setting, using the readout scheme. Further, we detect entanglement between two spatially distinct regions.

This thesis provides an overview over the theoretical and experimental details necessary to reach a high level of control for the detection of squeezing and entanglement. We start by giving a brief introduction of the theoretical concepts that describe our experimental platform, which is a spin-1 ^{87}Rb Bose-Einstein condensate. There, we focus on the important dynamics of spin-changing collisions and visualize the time evolution using semi-classical truncated Wigner simulations. To gain an overview over the experiment we summarize its setup including the imaging setup and manipulations to the ^{87}Rb atom cloud in chapter 3. In this chapter, we also focus on the newly implemented arbitrary waveform generator setup. In chapter 4,

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we explain the readout scheme as well as its calibration. Finally, chapter 5 provides a summary of our analysis method also including truncated Wigner simulations following the experimental readout sequence. This chapter ends with the presentation of our experimental results of squeezing and the detection of entanglement between the left and right half of our system.

2 Theoretical background

The experiments are performed in a ^{87}Rb Bose-Einstein condensate (BEC). With ^{87}Rb being in its electronic groundstate the two hyperfine manifolds $F = 1$ and $F = 2$ are utilized as our experimental platform. In the following chapter, we introduce the theoretical concepts to understand the processes and dynamics in such ultracold systems. We will see, that the $F = 1$ platform in ^{87}Rb is a spin-1 system. To obtain a full operator basis and understanding for the spin-1 case, we use the key concepts from the widely discussed spin-1/2 system [20]. Later, we extend the single particle description to a collective spin system, where interactions give rise to the phenomena of squeezing and entanglement.

2.1 Spin-1

To describe a spin-1 system it can be useful to study the spin-1/2 system and to use the same concepts for the treatment of the spin-1 system. In the literature the spin-1/2 system is widely discussed [20] and well known. Thus, it will not be discussed in this thesis. However, the representation of a spin-1/2 system on the so-called Bloch-sphere will prove useful, as similar representations exist for the spin-1 system. The Bloch-sphere, is spanned by the eigenvectors of the spin-1/2 operators $\hat{S}_i = \hbar\hat{\sigma}_i/2$, with $i \in \{x, y, z\}$, $\hat{\sigma}_i$ being the Pauli matrices and $\hbar$ the reduced Planck's constant. This yields a sphere as shown in figure 2.1. Here, the eigenstates of $\hat{S}_z$, i.e. $|\uparrow\rangle = (1, 0)^T$ and $|\downarrow\rangle = (0, 1)^T$, are on the north and south pole of the sphere. A pure spin-1/2 state is described by a point on the sphere and is characterized by $|\psi_{1/2}\rangle = e^{i\phi/2} \cos \frac{\theta}{2} |\uparrow\rangle + e^{-i\phi/2} \sin \frac{\theta}{2} |\downarrow\rangle$, with $\theta \in [0, \pi]$ and $\phi \in [0, 2\pi]$. Any unitary operation is described by a rotation on the sphere. For example, the eigenstate of $\hat{S}_z$, $|\psi_i\rangle = |\uparrow\rangle = (1, 0)^T$ is rotated around the y -axis onto the equator by the unitary transformation $\hat{U} = e^{-i\pi\hat{S}_y/2}$.

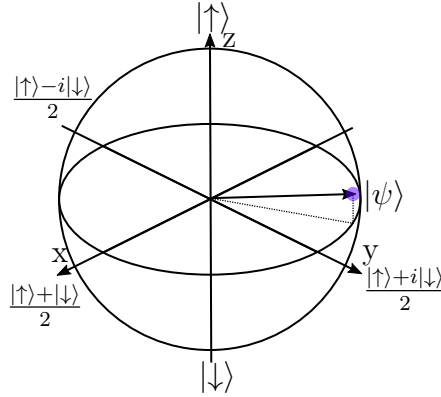


Figure 2.1: Bloch-sphere: The crossing points of the Bloch-sphere with the axes are given by the eigenstates of the spin operators. The eigenstates of $\hat{S}_z$ are positioned on the poles. A pure single particle state is represented by a point on the sphere as depicted for $|\psi\rangle$. Unitary operations are described by rotations on the sphere.

2 Theoretical background

Expanding the description of a spin-1/2 system by another component, yields for a general spin-1 state the following spinor

$$\begin{aligned} |\psi\rangle &= \sqrt{N_{+1}}e^{i\phi_L/2} | +1\rangle + \sqrt{N_0}e^{i\phi_0} | 0\rangle + \sqrt{N_{-1}}e^{-i\phi_L/2} | -1\rangle \\ &= \begin{pmatrix} \sqrt{N_{+1}}e^{i\phi_L/2} \\ \sqrt{N_0}e^{i\phi_0} \\ \sqrt{N_{-1}}e^{-i\phi_L/2} \end{pmatrix}. \end{aligned} \quad (2.1)$$

Here, we neglected an overall global phase and $\phi_L = \phi_{+1} - \phi_{-1}$ denotes the Larmor phase, which describes the phase difference between the states $|\pm 1\rangle$. The coefficients $\sqrt{N_i}$ and $i \in \{0, \pm 1\}$ are the real amplitudes, with N_i being the probability to find the particle in the state $|i\rangle$. Due to the normalization the coefficients are constrained by $\sum_i N_i = 1$. For the spin-1 system there is an additional phase, the so called Spinor phase $\phi_S = \phi_0 - (\phi_{-1} + \phi_{+1})/2$. It describes the relative phase between the state $|0\rangle$ and the states $|\pm 1\rangle$.

To map our experimental platform, the $F = 1$ hyperfine manifold of a ^{87}Rb Bose-Einstein condensate, to the spin-1 description, we identify the three magnetic substates $m_F = \{0, \pm 1\}$ with the components of the spin-1 spinor, cf. equation (2.1).

2.1.1 Spin-1 operators

In the spin-1/2 system the spin operators including the identity allow to describe any unitary transformation and span a operator basis. However, in the spin-1 case with its additional degrees of freedom these spin operators are not sufficient anymore. A possible choice for a complete set of operators are the so-called spin and spin-quadrupole operators [14]. In matrix notation, using the basis as in equation (2.1), the spin operators are given as

$$\hat{S}_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \quad \hat{S}_y = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \hat{S}_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad (2.2)$$

These operators fulfill the SU(3) commutation relation $[\hat{S}_i, \hat{S}_j] = i\epsilon_{ijk}\hat{S}_k$.

The spin quadrupole operators are defined via the relation [21, 22]

$$\hat{Q}_{ij} = \hat{S}_i\hat{S}_j + \hat{S}_j\hat{S}_i - \frac{4}{3}\delta_{ij}\mathbb{I}_3. \quad (2.3)$$

Here, δ_{ij} is the Kronecker Delta, $\mathbb{I}_3$ the identity and $i, j \in \{x, y, z\}$. Inserting the spin-1 operators $\hat{S}_i$ given in equation (2.2) into the above definition yields the quadrupole operators. Here, we give the matrix notation of the two relevant

quadrupole operators used in this thesis

$$\hat{Q}_{yz} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & i \\ 0 & i & 0 \end{pmatrix}, \quad \hat{Q}_0 = -\frac{1}{3}\mathbb{I}_3 - \hat{Q}_{zz} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}. \quad (2.4)$$

The definition in equation (2.3) would give us 6 quadrupole operators which would lead to 9 operators in total, when considering the three spin operators from above. However, this would result in an overcomplete basis. Thus, we select the 5 quadrupole operators, that are given in appendix A.1. These operators are the generators of rotations and allow to describe any unitary transformation. Further, they are used to characterize any state as decomposing the density matrix shows [14]

$$\hat{\rho} = \frac{1}{3}\mathbb{I}_3 + \sum_i s_i \hat{S}_i + \sum_j q_j \hat{Q}_j. \quad (2.5)$$

2.1.2 The spin-nematic space

From the spin-1/2 case, we know that we can represent a pure state as a point on the surface of the so-called Bloch-sphere, cf. figure 2.1. There, any rotation is generated by the spin operators and visualized by a rotation on the sphere. The rotation axis is defined by the generating spin operator. To visually describe a spin-1 state we introduce the spin sphere, which is spanned by the expectation values of the spin-1 operators $\{S_x, S_y, S_z\}$. This is comparable to the spin-1/2 case. Here, the corresponding spin operators are also the generators of rotation on the sphere. However, for the spin-1 system the spin sphere is not sufficient. In contrast to the spin-1/2 system, even a pure state can be located inside the spin-1 sphere, e.g. $|\psi\rangle = (0, 1, 0)^T$. This state has a vanishing mean spin in all directions $\langle\psi|\hat{S}_i|\psi\rangle = 0$, $i \in \{x, y, z\}$ and thus is located inside the sphere. To overcome this ambiguity caused by the expectation value of the spin operators we consider the quadrupole operators defined in equation (2.3).

With these, we are now able to define subsets consisting of three spin-1 operators $\{\hat{O}_1, \hat{O}_2, \hat{O}_3\}$ and each triplet obeying the commutation relation $[\hat{O}_i, \hat{O}_j] = i\epsilon_{ijk}\hat{O}_k$. In general several triplets can be found. However, for the following discussion we choose the triplet consisting of the three operators $\{\hat{S}_x, \hat{Q}_{yz}, \hat{Q}_0\}$ [21]. We make this choice, as we will measure the observables S_x and Q_{yz} later on. Their commutation relations are explicitly given by

$$\begin{aligned} [\hat{Q}_0, \hat{Q}_{yz}] &= 2i\hat{S}_x, & [\hat{S}_x, \hat{Q}_0] &= 2i\hat{Q}_{yz} \\ [\hat{Q}_{yz}, \hat{S}_x] &\approx 2i\hat{Q}_0. \end{aligned} \quad (2.6)$$

Thus, we can map this subspace onto a sphere, that is spanned by the expectation values $\{S_x, Q_{yz}, Q_0\}$. This is the so-called spin-nematic sphere [14, 21]. To visualize

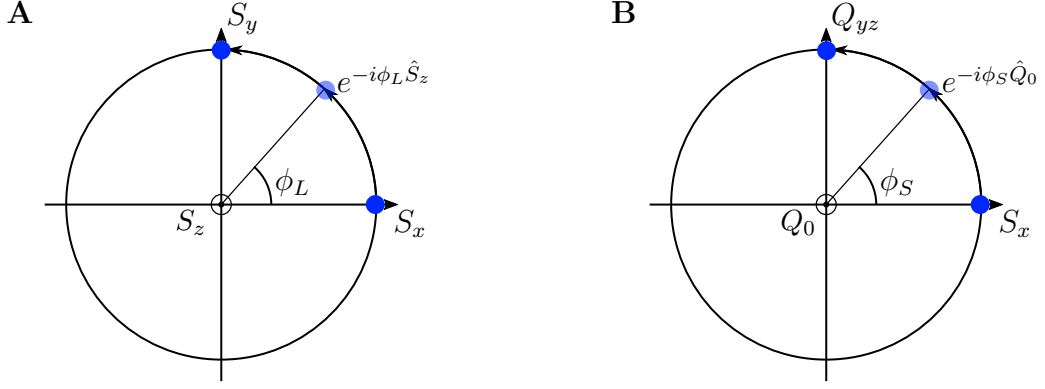


Figure 2.2: Spin sphere and spin-nematic subspace (A) On the spin sphere an initial eigenstate of S_x is rotated into an elongated spin in y direction by the unitary rotation $e^{-i\pi\hat{S}_z/2}$. This transformation imprints a Larmor phase onto the state. (B) On the spin-nematic sphere the unitary operation $e^{-i\phi_S\hat{Q}_0}$ introduces a quadrupole moment by evolving the Spinor phase.

the difference between the spin and spin-nematic sphere, we take a look at the equatorial plane, spanned by $\{S_x, S_y\}$ and $\{S_x, Q_{yz}\}$ for the spin and spin-nematic sphere, respectively. These planes are shown in figure 2.2. Starting in both cases with the eigenstate of $\hat{S}_x$, the unitary operation $e^{-i\phi_L\hat{S}_z}$ changes the Larmor phase by ϕ_L on the spin-plane. After a rotation of $\pi/2$ the state becomes an eigenstate of $\hat{S}_y$. On the spin-nematic plane the unitary transformation generated by $\hat{Q}_0$, i.e. $e^{-i\phi_S\hat{Q}_0/2}$, advances the Spinor phase by ϕ_S , where an additional factor of 2 is coming from the commutation relation. A rotation builds up a quadrupole moment and an evolution of $\pi/2$ turns the initial state into an eigenstate of Q_{yz} . Thus, depending on the ongoing dynamics each visualization has different advantages.

2.1.3 Collective spin

In the previous paragraphs we discussed the spin-1 operators and the representation of a spin-1 state on the spin- and spin-nematic sphere for a single particle state. However, our experiments are performed in a Bose-Einstein condensate (BEC) consisting of $\sim 10^4$ atoms. As we cannot resolve single particles and having this relatively large number of particles we make use of the properties of a BEC and define a collective operator $\hat{O}_{\text{coll}} = \sum_{k=1}^N \hat{O}_k$. Here, $\hat{O}_k$ is the single-particle spin-1 operator acting on the particle k . It proves useful to express these collective operators in second quantization using the Jordan-Schwinger map [23]. For example for $\hat{S}_x$ we get in

second quantization

$$\begin{aligned}\hat{S}_x &= (\hat{a}_{+1}^\dagger \quad \hat{a}_0^\dagger \quad \hat{a}_{-1}^\dagger) \hat{\mathcal{S}}_x \begin{pmatrix} \hat{a}_{+1} \\ \hat{a}_0 \\ \hat{a}_{-1} \end{pmatrix} \\ &= \frac{1}{\sqrt{2}} \hat{a}_0^\dagger (\hat{a}_{+1} + \hat{a}_{-1}) + \frac{1}{\sqrt{2}} (\hat{a}_{+1}^\dagger + \hat{a}_{-1}^\dagger) \hat{a}_0 ,\end{aligned}\tag{2.7}$$

where $\hat{\mathcal{S}}_x$ denotes the matrix representation, cf. equation (2.2), and $\hat{a}_{m_F}^{(\dagger)}$ are the creation and annihilation operator for the magnetic substate m_F , respectively. These operators obey the bosonic commutation relation $[\hat{a}_m, \hat{a}_n^\dagger] = \delta_{mn}$. Following this scheme, we can express the other spin-1 operators in second quantization. It can be checked, that the commutation relations introduced before do not change in second quantization. Thus, we can define the corresponding spin-sphere ($\{S_x/N, S_y/N, S_z/N\}$) and spin-nematic sphere ($\{S_x/N, Q_{yz}/N, Q_0/N\}$), where N denotes the total atom number. The rescaling with the atom number N ensures the representation on a unit sphere. As from now on we discuss collective spin states, we will not write the normalization to the atom number explicitly.

2.1.4 Coherent state

Usually, the starting point of our experiments is a coherent polar state. In this state all particles occupy the same $m_F = 0$ state and we can express this state with $|\psi_P\rangle = (|0\rangle)^{\otimes N}$. Evaluating the spin yields a vanishing spin S_i in all directions. However, this state features minimal fluctuations in the $S_x - S_y$ plane and no fluctuations in z -direction, see [14] for more details. By applying a $\pi/2$ rotation around S_y on the spin sphere, i.e. the unitary rotation $e^{-i\pi\hat{S}_y/2}$, we transform the polar state into a so-called transversal polar state. This rotates the fluctuations into the $z - y$ plane. From the mapping to second quantization we know, that $\hat{S}_z = \hat{a}_{+1}^\dagger \hat{a}_{+1} - \hat{a}_{-1}^\dagger \hat{a}_{-1}$. Using $\hat{n}_i = \hat{a}_i^\dagger \hat{a}_i$ with $i \in \{0, \pm 1\}$, we can express $\hat{S}_z$ via $\hat{S}_z = \hat{n}_{+1} - \hat{n}_{-1}$. Thus, by detecting the atom number difference $N^- = N_{-1} - N_{+1}$ after the rotation we are able to extract S_x of the initial polar state. This allows to determine the fluctuations of the polar state. The underlying probability distribution for the measurement of N^- is given by a binomial distribution [14]

$$P(N^-) = \binom{N}{\frac{N}{2} + N^-} p_{+1}^{\frac{N}{2} + N^-} p_{-1}^{\frac{N}{2} - N^-} .\tag{2.8}$$

Here, $p_{+1} = p_{-1} = 0.5$ denote the probability to find a particle in the state $|\pm 1\rangle$. For large N the binomial distribution can be approximated by a Gaussian distribution with mean $\langle N^- \rangle = 0$ and variance $\Delta^2 N^- = N$. Thus, the N particle polar state is represented in the equatorial plane (x - y) of the generalized spin sphere, by a gaussian distribution.

2.2 Spin mixing

Having an ensemble of atoms in our BEC gives rise to internal interactions and dynamics. Focusing on the spin-spin interaction will show the emergence of squeezed states and entanglement.

2.2.1 Interaction Hamiltonian

Having a dilute spinor gas in our experiment, allows to take only binary contact interactions into account. Further, assuming spherical symmetric collisions and therefore a conservation of the total angular momentum, the interactions can be described by s-wave scattering. Considering the trapping potential and the external magnetic field, the Hamiltonian $\hat{\mathcal{H}}$ can be divided into three parts [24]:

$$\hat{\mathcal{H}} = \int d^3r \left(\hat{\mathcal{H}}_0 + \hat{\mathcal{H}}_B + \hat{\mathcal{H}}_{\text{int}} \right) . \quad (2.9)$$

The first part $\hat{\mathcal{H}}_0$ describes the single particle kinetic and potential energy in the confining trap $V_{\text{trap}}(\mathbf{r})$

$$\hat{\mathcal{H}}_0 = \sum_i \hat{\psi}_i^\dagger(\mathbf{r}) \left(\frac{-\hbar^2 \nabla^2}{2m} + V_{\text{trap}}(\mathbf{r}) \right) \hat{\psi}_i(\mathbf{r}) , \quad (2.10)$$

with m being the mass and $i = 1, \dots, N$, where N denotes the total atom number and $\hat{\psi}(\mathbf{r})$ is the single mode wavefunction. Due to the application of an external magnetic field we have to consider the energy shifts caused by the linear and quadratic Zeeman shift, which gives us the second part

$$\hat{\mathcal{H}}_B / \hbar = p \hat{S}_z(\mathbf{r}) + q (\hat{n}_1(\mathbf{r}) + \hat{n}_{-1}(\mathbf{r})) . \quad (2.11)$$

where $\hat{n}_{m_F}$ denotes the number operator for the the magnetic substate m_F . Here, p stands for the Linear Zeeman frequency shift and q for the quadratic Zeeman frequency shift, which depend on the external magnetic field. The third part $\hat{\mathcal{H}}_{\text{int}}$, denotes the energy shift due to inter particle interactions and is given as

$$\hat{\mathcal{H}}_{\text{int}} = : \left(\tilde{c}_0 \hat{n}^2(\mathbf{r}) + \tilde{c}_1 \left(\hat{S}_x^2(\mathbf{r}) + \hat{S}_y^2(\mathbf{r}) + \hat{S}_z^2(\mathbf{r}) \right) \right) : , \quad (2.12)$$

where, $:$ denotes normal ordering and $\hat{n} = \sum_i \hat{n}_i$ is the total density operator, with $i \in \{0, \pm 1\}$. The coupling constants are given as [14]

$$\begin{aligned} c_0 &= \frac{4\pi\hbar^2}{m} \frac{a_0 + 2a_2}{3} , \quad \text{and} \\ c_1 &= \frac{4\pi\hbar^2}{m} \frac{a_2 - a_0}{3} . \end{aligned} \quad (2.13)$$

Further considering the mode overlap χ in the single mode approximation, i.e. all states occupy the same spatial mode ψ_{sm} , we find $\chi = \int d\mathbf{r} |\psi_{\text{sm}}|^4$. This leads to the modified coupling constants $\tilde{c}_i = \chi c_i$.

In the case of the $F = 1$ hyperfine manifold of ^{87}Rb the spin dependent coupling constant c_1 is negative. Therefore, the energy in equation (2.12) is minimized if the spin of all atoms is aligned. Thus, this state is called ferromagnetic.

2.2.2 Spin mixing Hamiltonian

In the previous paragraph we have looked at the overall Hamiltonian and found that we have a quadratic (non-linear) spin dependent part in the interaction Hamiltonian $\hat{\mathcal{H}}_{\text{int}}$. In the following, we will look into this part, as it causes spin-mixing dynamics between the magnetic substates. To understand the effect we insert the representation of the operators in second quantization [23, 25]. We then get for the spin dependent part:

$$\begin{aligned} \hat{\mathcal{H}}_{\text{Spin}} &= \tilde{c}_1 : \left(\hat{S}_x \hat{S}_x + \hat{S}_y \hat{S}_y + \hat{S}_z \hat{S}_z \right) : \\ &= 2\tilde{c}_1 \hat{n} \end{aligned} \quad (2.14)$$

$$+ \tilde{c}_1 \left(\hat{n}_0 - \frac{1}{2} \right) (\hat{n}_{+1} + \hat{n}_{-1}) \quad (2.15)$$

$$+ \tilde{c}_1 : (\hat{n}_{+1} - \hat{n}_{-1})^2 : \quad (2.16)$$

$$+ 2\tilde{c}_1 \left(\hat{a}_0^\dagger \hat{a}_0^\dagger \hat{a}_{+1} \hat{a}_{-1} + \hat{a}_{+1}^\dagger \hat{a}_{-1}^\dagger \hat{a}_0 \hat{a}_0 \right) . \quad (2.17)$$

The first (2.14) and second (2.15) term in the above equation introduce an overall energy offset. Thus they do not play a role for internal dynamics and therefore can be neglected. Further, the third term (2.16) can be neglected for our experiments as we make sure, that the imbalance between the states $m_F = \pm 1$ is zero. The last term (2.17) is important as it gives rise to internal dynamics. It transfers atom pairs from the state $m_F = 0$ to the states $m_F = \pm 1$ and vice versa [26].

Producing atom pairs between states $m_F = 0 \leftrightarrow m_F = \pm 1$ does not change the energy offset under the first order Zeeman effect, as the transfer does not change S_z and the energy difference is $\propto \hat{S}_z$, cf. equation (2.11). The energy needed to produce a particle in the state $m_F = -1$ is gained by producing a particle in the state $m_F = +1$. Therefore, the relevant energy scale is the second-order Zeeman shift, which depends on the sidemode population. Following these arguments, we can express the Hamiltonian describing the internal dynamics by

$$\mathcal{H}/\hbar = -\frac{q}{2} \hat{Q}_0 + \frac{\tilde{c}_1}{2} : \left(\hat{S}_x^2 + \hat{S}_y^2 + \hat{S}_z^2 \right) : , \quad (2.18)$$

where we neglected the first-order Zeeman shift and q denotes the energy shift of the second-order Zeeman effect. Thus, tuning q allows us to initiate spin-mixing

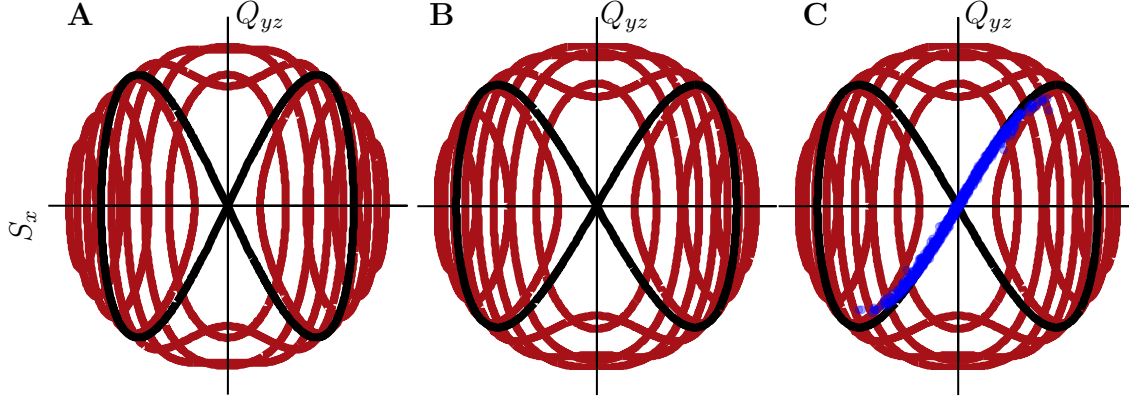


Figure 2.3: Phase-space: We evaluated the equipotential lines from the spin-mixing Hamiltonian, cf. equation (2.18), for $(\tilde{c}_1, q) = (-2, 1.5)$ (**A**) and $(\tilde{c}_1, q) = (-2, 2)$ (**B**). With increasing q the covered extension of the black separatrix $q = N\tilde{c}_1$ gets larger. (**C**) For the values from (B) we added a time evolution from a polar state of a numerical simulation.

dynamics. Making q smaller, the energy difference, between the initially populated state $m_F = 0$ and the states $m_F = \pm 1$ gets smaller and thus, spin-changing collisions get more likely. The arising equipotential lines from the spin-mixing Hamiltonian are shown in figure 2.3 for different value pairs of $\tilde{c}_1$ and q .

Further, the spin-mixing dynamics create a so-called two-mode squeezed vacuum state, that is highly entangled. It has been discussed in the literature, that these dynamics can be compared with four-wave mixing [21, 27].

2.2.3 Visualization of spin dynamics using truncated Wigner simulations

In the previous chapter we introduced interactions in the Bose gas and specifically looked at spin-mixing dynamics. These dynamics lead to squeezing and create a highly entangled state.

A method to visualize the dynamics of the system are so-called truncated Wigner simulations. This simulation capture well the short time dynamics for large atom numbers, e.g. $\sim 10^4$ [14, 28].

To implement a truncated Wigner simulation we start by defining our initial state as

$$\psi_S = \sqrt{N} \begin{pmatrix} \zeta_{+1} \\ \zeta_0 \\ \zeta_{-1} \end{pmatrix}, \quad (2.19)$$

with the random complex numbers ζ_i , $i = 0, \pm 1$ and $\sqrt{N}$ representing our atom number. The $X_{\pm 1} = \sqrt{N}\zeta_{\pm 1}$ are chosen such that they have a vanishing mean

$\langle X_{\pm 1} \rangle = 0$ and a variance $\Delta^2 X_{\pm 1} = 1/2$. This is also known as the quantum one-half [14]. Calculating the mean spin $\langle \psi_S | \hat{S}_x | \psi_S \rangle = 0$ and variance $\Delta^2 S_x = N$, yields the same result as expected for a polar state, cf. section 2.1.4.

Using the quantum mechanical time evolution $d\hat{a}_m^{(\dagger)} / dt = i [\hat{\mathcal{H}}, \hat{a}_m^{(\dagger)}] / \hbar$ and inserting the classical approximation

$$\begin{aligned}\hat{a}_{+1}^{(\dagger)} &= \sqrt{N} \zeta_{+1}^{(*)} \\ \hat{a}_0^{(\dagger)} &= \sqrt{N} \zeta_0^{(*)} \\ \hat{a}_{-1}^{(\dagger)} &= \sqrt{N} \zeta_{-1}^{(*)}\end{aligned}\tag{2.20}$$

yield the classical equation of motion [14]

$$\begin{aligned}\frac{d\zeta_{+1}}{dt} &= -i2\pi N \tilde{c}_1 [(|\zeta_{+1}|^2 + |\zeta_0|^2 - |\zeta_{-1}|^2) \zeta_{+1} + \zeta_0^2 \zeta_{-1}^*] \\ \frac{d\zeta_0}{dt} &= i2\pi q \zeta_0 - i2\pi N \tilde{c}_1 [(|\zeta_{+1}|^2 + |\zeta_{-1}|^2) \zeta_0 + 2\zeta_{+1} \zeta_{-1} \zeta_0^*] \\ \frac{d\zeta_{-1}}{dt} &= -i2\pi N \tilde{c}_1 [(-|\zeta_{+1}|^2 + |\zeta_0|^2 + |\zeta_{-1}|^2) \zeta_{-1} + \zeta_0^2 \zeta_{+1}^*] ,\end{aligned}\tag{2.21}$$

with ζ_m , $m \in \{0, \pm 1\}$ being a complex number and the normalization $|\zeta_m|^2 = N_m/N$. These equations can be numerically solved to study the dynamics.

The simulation yields complex numbers thus we are able to directly extract the expectation value of the observables S_x and Q_{yz} using

$$\begin{aligned}\langle S_x \rangle &= \langle \psi_f | \mathcal{S}_x | \psi_f \rangle \\ \langle Q_{yz} \rangle &= \langle \psi_f | \mathcal{Q}_{yz} | \psi_f \rangle .\end{aligned}\tag{2.22}$$

Here, $\mathcal{S}_x$ and $\mathcal{Q}_{yz}$ denote the matrix representation given in equation (2.2) and (2.4), respectively and ψ_f is the vector after the time evolution.

Simulation of dynamics

We prepare a polar state ($T_{\text{Evo}} = 0$ ms), cf. equation (2.19), and let it evolve for 200 ms. We choose $q_{\text{eff}} = -N\tilde{c}_1 = 2$ Hz for the free parameters, as this is comparable to our experiment. We repeat this simulation 1000 times with a fixed atom number of $N = 2 \times 10^4$. Here, the effect of the atom number is the relative size of the fluctuations compared with the mean value. As depicted in equation 2.19 the fluctuations of the states ± 1 are fixed to $1/2$. Thus, changing N and keeping $\Delta^2 X_i = 1/2$ fixed, changes the relative size. From each repetition we get a point in the phase space spanned by S_x and Q_{yz} . In figure 2.4 we exemplary show the phase space after $T_{\text{evo}} = 0, 100, 200$ ms. We normalized the observables by the mean atom number $\langle N \rangle$. In figure 2.4 A we clearly see the initial polar state, that

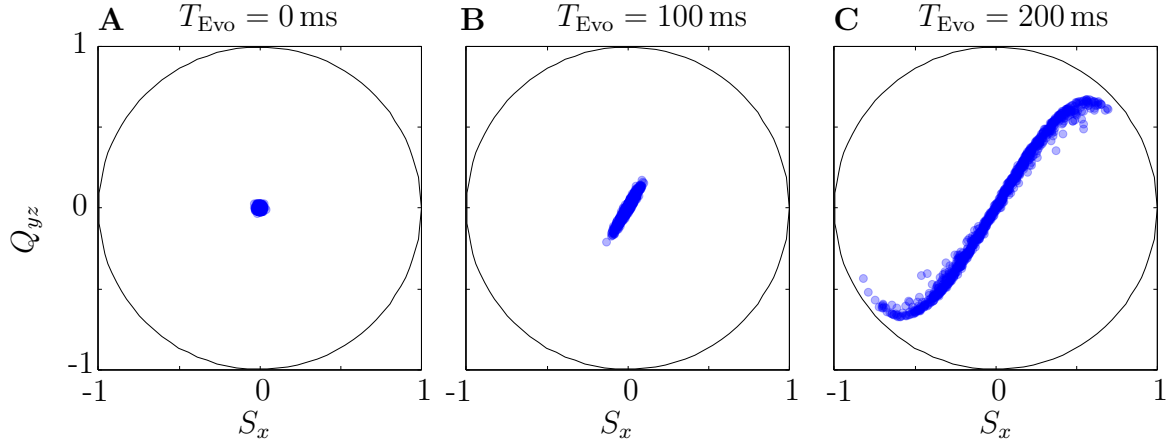


Figure 2.4: Truncated Wigner simulations We used truncated Wigner simulations to simulate the spin-mixing dynamics for evolution times $T_{\text{Evo}} = 0, 100$ and 200 ms, with $N = 20.000$ atoms. **A** Initial coherent state. **B** The phase space of the observables S_x and Q_{yz} after $T_{\text{Evo}} = 100$ ms is shown. The state is still in the Gaussian regime and the squeezing ellipses can be clearly seen. **C** Phase space evolution after $T_{\text{Evo}} = 200$ ms. The state follows the classical contours [14], cf. figure 2.3

has an isotropic distribution and thus constant fluctuations in any direction, as we expected. However, the second plot, i.e. after $T_{\text{evo}} = 100$ ms, features a very interesting behavior. Orthogonal to the long axis, which evolves along the separatrix and where we have enhanced fluctuations, we see reduced fluctuations. Yet, this state can still be characterized with a Gaussian distributions in any direction. For later times, the state becomes non Gaussian and follows the classical trajectories, which form for the chosen parameters a “S”-shape, cf. figure 2.3 B and C. This behavior is illustrated in subplot 2.4 C, where we show the evolution after $T_{\text{Evo}} = 200$ ms. Nevertheless, the truncated Wigner simulations enable us to gain some intuition about the behavior of the spin-mixing dynamics.

2.3 Entanglement

Having introduced the spin-mixing process to obtain squeezing we now turn to the theoretical description of squeezing as a non-classical phenomenon.

2.3.1 Squeezing

To study the effect of squeezing it is useful to investigate the harmonic oscillator. The treatment of the harmonic oscillator allows us to understand the basic concepts of squeezing and apply it to spin-nematic squeezing later. The Hamiltonian of the

harmonic oscillator for a given frequency ω reads [29]

$$\mathcal{H} = \hbar\omega \left(\hat{n} + \frac{1}{2} \right) = \hbar\omega \left(\hat{a}^\dagger \hat{a} + \frac{1}{2} \right), \quad (2.23)$$

where $\hat{a}^{(\dagger)}$ are the creation and annihilation operator with $[\hat{a}, \hat{a}^\dagger] = 1$ and $\hat{n}$ is the number operator. For the ground state ($n = 0$) the wavefunction in the position basis is given as

$$\psi_0(X) = \frac{1}{\pi^{1/4}} e^{-X^2/2}. \quad (2.24)$$

The Fourier transformation then yields the wavefunction in momentum space

$$\psi_0(P) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{iPX} \psi_0(X) dX = \frac{1}{\pi^{1/4}} e^{-P^2/2}. \quad (2.25)$$

Here, we used the dimensionless form with the commutation relation $[\hat{X}, \hat{P}] = i$. Further, calculating the variance of X and P respectively, results in $\langle \psi_0 | \Delta^2 X | \psi_0 \rangle = \langle \psi_0 | \Delta^2 P | \psi_0 \rangle = 1/2$, where both operators have a symmetric distribution. Thus, the ground state saturates the uncertainty relation $\Delta^2 X \cdot \Delta^2 P \geq 1/4$. Having briefly summarized the basics of the harmonic oscillator, we now introduce the squeezing parameter R , which transform the vacuum state $|\psi_0\rangle$ into a squeezed state $|\text{sq}\rangle_R$. Here, we linearly transform the space and momentum observables

$$X \rightarrow RX \quad \text{and} \quad P \rightarrow P/R, \quad (2.26)$$

and we get for the corresponding wavefunction

$$\psi_R(X) = \frac{\sqrt{R}}{\pi^{1/4}} e^{-(RX)^2/2}, \quad (2.27)$$

and in momentum space

$$\psi_R(P) = \frac{1}{\pi^{1/4} \sqrt{R}} e^{-(P/R)^2/2}. \quad (2.28)$$

As before, we calculate the variance, which now yields

$$\Delta^2 X = \frac{1}{2R^2} \quad \text{and} \quad \Delta^2 P = \frac{R^2}{2}. \quad (2.29)$$

This state still saturates the uncertainty relation. But for $R \neq 1$ their variance differs. Let $R > 1$, then the variance for X decreases, which results in a position squeezed state, as the variance for $|\text{sq}\rangle_R$ is smaller compared to the vacuum state

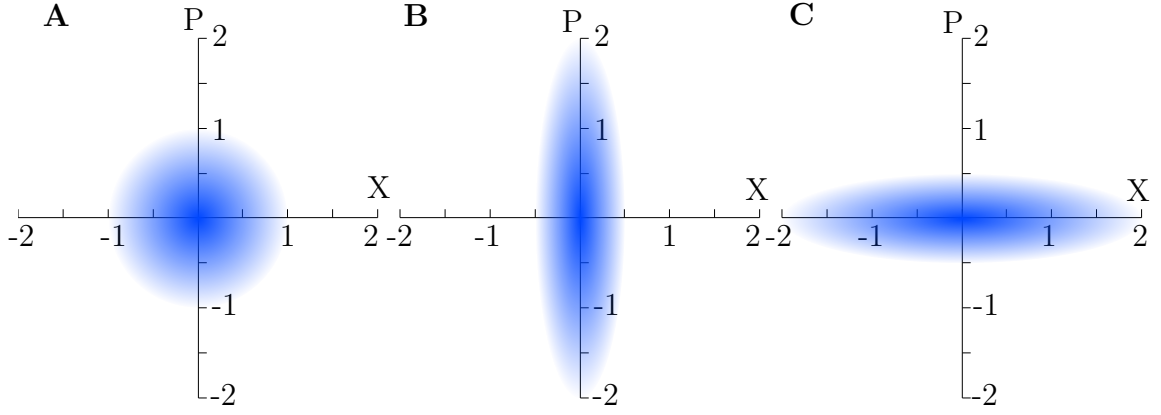


Figure 2.5: Single mode squeezing: Phase space distribution for different types of single mode squeezing. (A) Vacuum state with isotropic distribution, which serves as a reference. (B) Position squeezed state, which has reduced fluctuations in space and resulting enhanced fluctuations in momentum ($R > 1$). (C) Momentum squeezed state, that has reduced fluctuations in the momentum direction ($R < 1$).

$|0\rangle$. Analogously, for $R < 1$ we get a momentum squeezed state. To understand the effect of squeezing one can investigate the phase space spanned by the two observables X and P . While for the vacuum state, we get a symmetric distribution, see figure 2.5 A, the distribution becomes elliptical for the position and momentum squeezed state, respectively (figure 2.5 B and C). The phase space volume stays constant under this linear transformation.

So far we looked at one squeezed mode, i.e. one oscillator. Now we want to investigate the so-called two-mode squeezed vacuum state, where two oscillators are involved. This state is treated similarly to the single mode case but shows in contrast new features. To understand the two-mode squeezed vacuum state we start by looking at its wavefunction. For two harmonic oscillators we get the tensor product of the two vacuum states $|0\rangle \otimes |0\rangle$, which is expressed in the position basis by

$$\psi_{00}(X_A, X_B) = \frac{1}{\sqrt{\pi}} e^{-X_A^2/2} \cdot e^{-X_B^2/2}, \quad (2.30)$$

where X_A and X_B are the position observables in the two oscillators. It is useful to rewrite the wavefunction in the following way

$$\psi_{00}(X_A, X_B) = \frac{1}{\sqrt{\pi}} e^{-(X_A - X_B)^2/4} \cdot e^{-(X_A + X_B)^2/4}. \quad (2.31)$$

This allows to identify the new observables $(X_A - X_B)/\sqrt{2}$ and $(X_A + X_B)/\sqrt{2}$, which obey a Gaussian distribution, from which a variance of $1/2$ can be directly inferred. However, this does not come as a surprise, as we created this two-mode

state from two uncorrelated vacuum states (A and B), that each have a variance of $1/2$ as we calculated before. Additionally, the behavior in momentum space can be obtained similarly and yields the same result.

As for the squeezed state, we now introduce the squeezing parameter R to obtain the two-mode squeezed vacuum state $|\text{TMSV}\rangle_R$, which reads

$$\psi_{00}(X_A, X_B) = \frac{1}{\sqrt{\pi}} e^{-(X_A+X_B)^2/(4R^2)} \cdot e^{-R^2(X_A-X_B)^2/4} . \quad (2.32)$$

Applying the Fourier transform yields the wavefunction in momentum space

$$\psi_{00}(P_A, P_B) = \frac{1}{\sqrt{\pi}} e^{-(P_A-P_B)^2/(4R^2)} \cdot e^{-R^2(P_A+P_B)^2/4} . \quad (2.33)$$

Thus, for $R > 1$ we get squeezing in the observable $(X_A - X_B)/\sqrt{2}$ as the variance is below $1/2$. While this implies correlations in the momentum measurement in system A and B , we further see anti-correlations in momentum space, i.e. the observable $(P_A + P_B)/\sqrt{2}$ is squeezed below the vacuum limit. Consistent with the uncertainty relation, we get enhanced fluctuations in the orthogonal observables, i.e. $(X_A + X_B)/\sqrt{2}$ and $(P_A - P_B)/\sqrt{2}$. The corresponding behavior is sketched in figure 2.6. Here, we show the wavefunction in real and momentum space, revealing strong correlations and anti-correlations, respectively. As a reference, we give the two-mode vacuum state in real space, see figure 2.6 A. However, if we consider each mode individually we do not find squeezing, and further find a variance which is greater than the variance of the vacuum state for all $R \neq 1$. More precisely, the variance is given as

$$\Delta^2 X_A = \Delta^2 X_B = \Delta^2 P_A = \Delta^2 P_B = \frac{1 + R^4}{4R^2} > \frac{1}{2} , \quad R \neq 1 . \quad (2.34)$$

Thus, having squeezing in the two-mode state and enhanced fluctuations in the individual modes is a strong sign for non-classical correlations, i.e. entanglement.

2.3.2 Spin-nematic squeezing

We have introduced squeezing in a general way by using the description of the quantum harmonic oscillator. Now, we want to extend this description which suits our experimental environment. Therefore we adapt the definition of spin-nematic squeezing given in [21, 30]. The main idea is, that for a coherent state the uncertainty relation of two orthogonal spin observables is saturated. Thus, for a spin squeezed state the former isotropic quasi-probability phase space distribution obtains a preferred direction, i.e. the variance in one direction decreases while the variance in the orthogonal direction increases, which results in an elliptical phase space distribution. A coherent state and a squeezed state are visualized in figure 2.4 A and B, respectively.

Thus, we start by investigating the coherent state, which features an isotropic

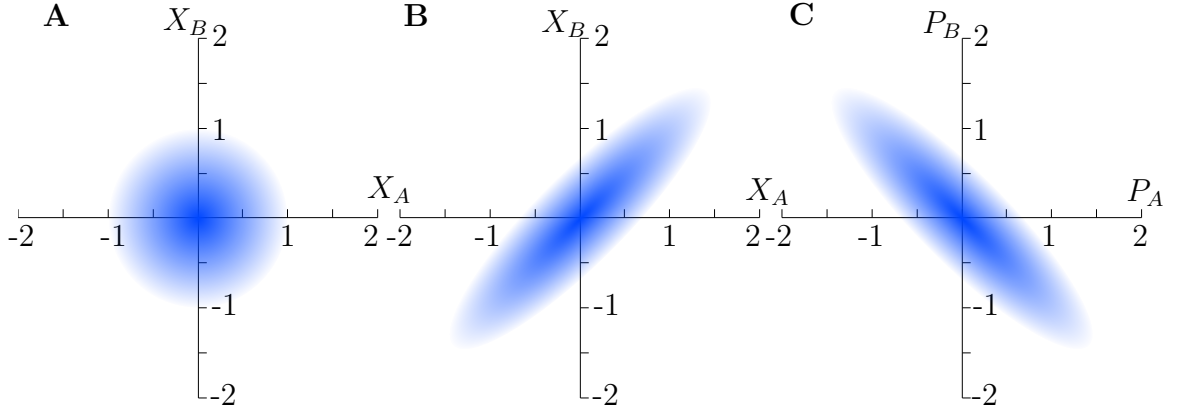


Figure 2.6: Two-mode squeezing: We show the wavefunction for two mode squeezing with the subsystems A and B . **(A)** Wavefunction representing the two-mode state. **(B)** and **(C)** While the spatial distribution shows strong correlation, the corresponding distribution in momentum space has strong anticorrelations, due to the two-mode squeezing.

quasi-probability distribution and has minimal uncertainty. Such a state is the polar state, which was introduced in section 2.1.4. The uncertainty relation for the relevant spin-1 operators $\hat{S}_x$ and $\hat{Q}_{yz}$ is obtained by inserting them into the general relation, which is given by [31]

$$\Delta^2 \hat{A} \Delta^2 \hat{B} \geq \frac{1}{4} \left| \langle [\hat{A}, \hat{B}] \rangle \right|^2. \quad (2.35)$$

With the relevant spin-1 operators we get

$$\Delta^2 \hat{S}_x \Delta^2 \hat{Q}_{yz} \geq \frac{1}{4} \left| \langle [\hat{S}_x, \hat{Q}_{yz}] \rangle \right|^2 = N^2. \quad (2.36)$$

Here, we make use of the sidemode population $\hat{N}^+$ of the states $|\pm 1\rangle$ being much smaller than the atom number N . The complete commutator is given as

$$[\hat{S}_x, \hat{Q}_{yz}] = i \left(2\hat{N}_0 - (\hat{a}_1^\dagger - \hat{a}_{-1}^\dagger)(\hat{a}_1 - \hat{a}_{-1}) \right). \quad (2.37)$$

In the experiment we work in the undepleted pump region, where only a few atom pairs (≈ 8) are created in the sidemodes. Having an atom number of $\approx 4 \times 10^4$ allows to neglect the term $(\hat{a}_1^\dagger - \hat{a}_{-1}^\dagger)(\hat{a}_1 - \hat{a}_{-1})$, which accounts for the imbalance in the sidemodes. Thus, the commutation relation can be approximated as $[\hat{S}_x, \hat{Q}_{yz}] \approx 2i\hat{N}$, which yields the result in the above equation.

Thus, we have a spin-nematic squeezed state if the variance in one observable is below the variance of the coherent state.

3 Experimental control

After the introduction of the theoretical concepts in the previous chapter, we now turn to the discussion of our experimental setup. Besides summarizing the geometrical setup we put an emphasis on the imaging and precise control of the experiment.

3.1 Experimental setup

Our experiments are performed in a Bose-Einstein condensate (BEC) of ^{87}Rb . Here, we focus on the trap geometry. More detailed information about the setup and cooling procedure can be found in [14, 19].

Our trap is a crossed-dipole trap, which is created by two far-red detuned (1030 nm), focused laser beams. These are referred to XdT and waveguide (WG). The setup is sketched in figure 3.1 A. For the trap frequencies we find $(\omega_{\parallel}, \omega_{\perp}) \approx 2\pi \cdot (50, 286)$ Hz of the XdT and $(\omega_{\parallel}, \omega_{\perp}) \approx 2\pi \cdot (2, 170)$ Hz of the WG potential [14, 19]. Here, $\omega_{\parallel}$ and $\omega_{\perp}$ denote the longitudinal and transversal direction, respectively. Coming from the cooling setup, we load the atoms into the crossed-dipole trap. There we can further cool down the sample by evaporative cooling, by lowering the depth of the dipole trap. This final step allows to tune the final atom number (usually 10,000 - 100,000 atoms) in the condensate. After the cooling and condensation procedure we get Bose-Einstein condensate, where all the atoms occupy the state $|F, m_F\rangle = |1, -1\rangle$. For our experiments, we then ramp up the power of the beams, to create a tightly confining harmonic trap (XdT), where we employ spin-mixing dynamics. Prior to the readout sequence, we switch off one of the confining beams and let the atom cloud expand in the remaining potential of the weakly focused waveguide.

Further, we usually apply a magnetic bias field of ≈ 1.5 G in our experiments. This is orientated along the z -axis, which we define as our quantization axis. The magnetic field gives rise to the Zeeman shift in the otherwise degenerated magnetic hyperfine states. To first order the Zeeman shift for a magnetic offset field $B = |\mathbf{B}|$ is given as $p = \mu_B g_F m_F B$, with μ_B the Bohr magneton and g_F the g-factor. For $F=1$ the g-factor is $g_{F=1} = -1/2$ and for $F=2$ $g_{F=2} = +1/2$. The energy splitting due to the first order Zeeman effect is sketched in figure 3.2 A, where also numerical values are given. The energy difference between the states $|1, \pm 1\rangle$ leads to an effective evolution of the Larmor phase. Further, we can expand the perturbation caused by the external magnetic field to second order, which gives rise to the quadratic Zeeman shift $q = (g_J - g_I)^2 \mu_B^2 / (16 \hbar^2 \omega_{HFS}) B^2$. The energy of the quadratic term is much smaller than the linear term and controls the spin-mixing dynamics, as we have seen in section 2.2.2. To see the effect, we transform to the co-rotating Larmor frame, which removes the linear Zeeman shift. Then, the quadratic term leads to a detuning of the side modes $m_F = \pm 1$ compared to the zero mode $m_F = 0$. Thus, the second order Zeeman shift leads to a Spinor phase evolution [19]. The corresponding energy scale is sketched in figure 3.2 B.

3 Experimental control

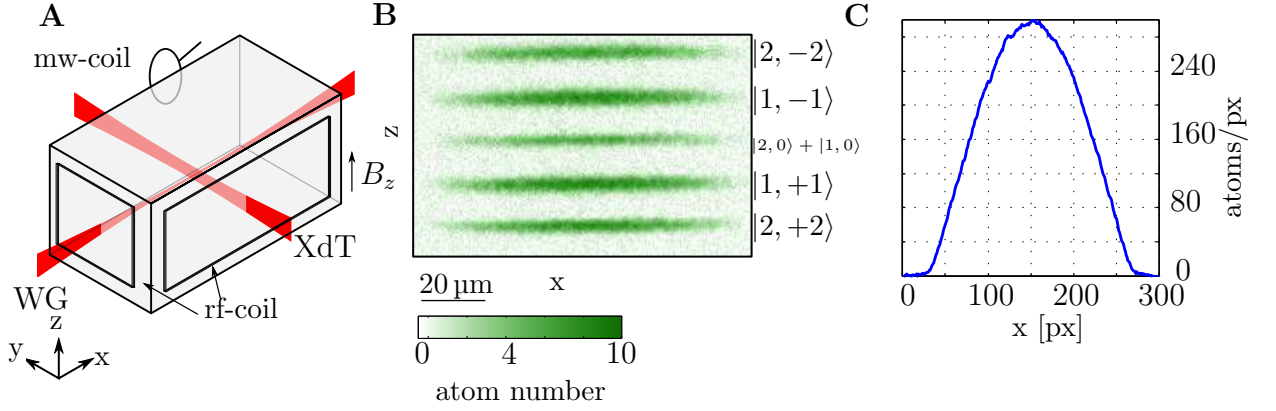


Figure 3.1: Experimental setup and atom cloud (A) Schematics of the experimental glass cell. (B) Here, an example image of the condensate after the expansion in the WG is shown. Before imaging we switch off the XdT and the atoms expand in the remaining WG potential. A Stern-Gerlach pulse prior to the imaging pulse spatially separates the magnetic substates along the z -axis. Imaging along the y -direction yields the shown clouds. Our imaging setup yield a combined picture of the $F = 1$ and $F = 2$ manifold. Due to our readout scheme we do not have atoms in $|F, m_F\rangle = |2, \pm 1\rangle$. (C) This plot shows an averaged atom density of all levels. Summing over the remaining z -extension (cf. B) gives a quasi 1-d distribution. It elucidates the harmonic trap geometry.

3.2 Imaging

To image the atomic densities we use high intensity absorption imaging [16]. The applied sequence allows to individually image the $F = 1$ and $F = 2$ hyperfine manifold [14]. We start our imaging sequence with switching off the XdT beam such that the atom cloud can expand in the remaining waveguide potential. After expanding the atomic cloud we apply a strong magnetic field gradient in z -direction to spatially separate the magnetic substates (Stern-Gerlach pulse). We first illuminate the atoms with light resonant to the $F = 2$ hyperfine manifold, i.e. resonant with an excited electronic state (D_2 light). We image the atoms using a CCD camera (Princeton Instruments PIXIS 1024BR) and further push them out of focus with the imaging light. Subsequently, repumper light ($F = 1 \rightarrow F = 2$) is added. This allows imaging the atoms in $F = 1$. The length of an imaging pulse is $\approx 15 \mu\text{s}$. For the squeezing measurements presented in this thesis we image the $F = 1$ and $F = 2$ hyperfine manifold in a single frame. To visualize we show a post processed absorption image in figure 3.1 B, where the individual magnetic substates are labeled. Imaging along the y direction yields the 2-d distribution shown in the figure. With this setup we loose the information stored in the state $|F, m_F\rangle = |2, 0\rangle$, as these atoms are overlapped with the atoms from the state $|1, 0\rangle$. However, we do not need this information in our further analysis. After imaging the atoms, we take two reference pictures without any atoms, which allow to determine the atomic densities from the measured intensities. To estimate the offset introduced by the camera, we further take a dark image without any light, after the imaging sequence.

However, usually both manifolds are imaged on two separate frames. This gives

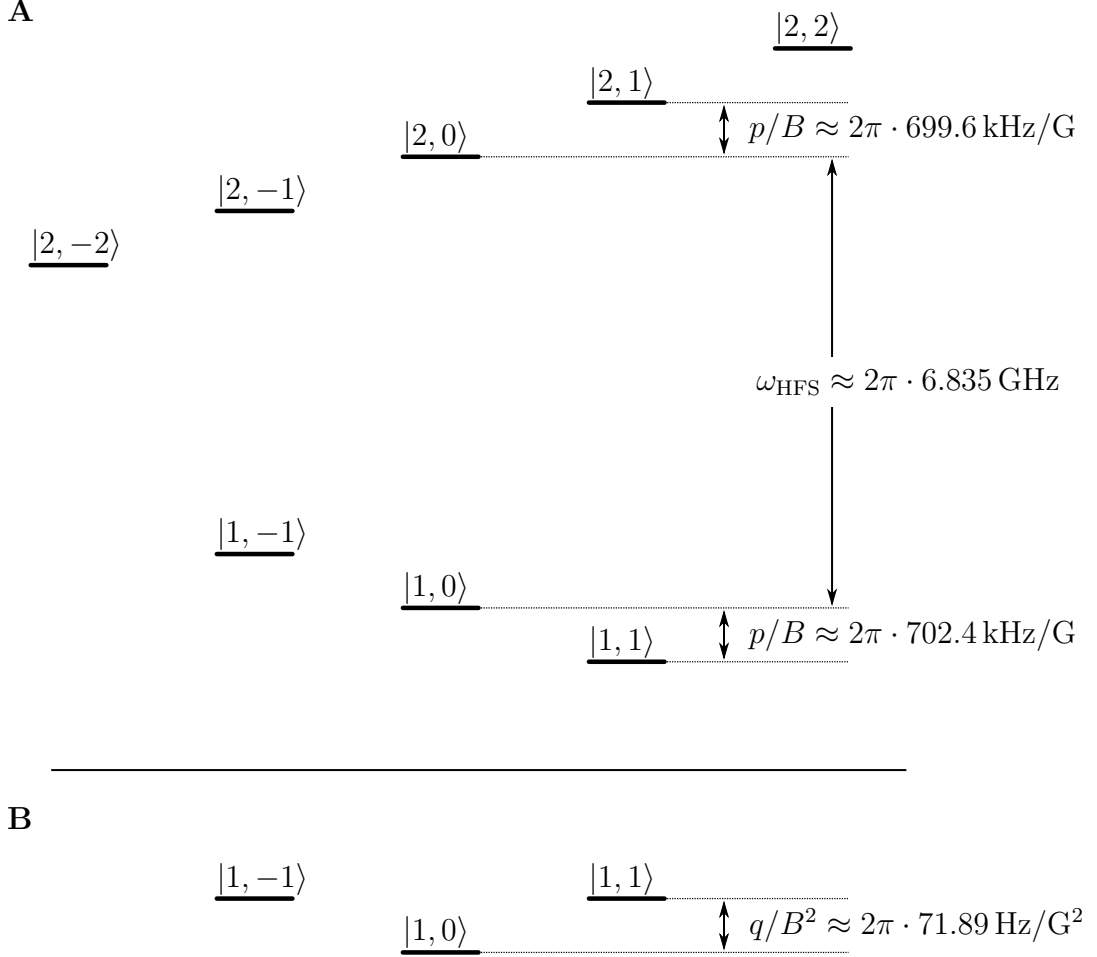


Figure 3.2: Energy spacings: Here, we give the schematic level structure and the energy spacings for the relevant levels in this thesis. The notations $|F, m_F\rangle$ is used. **(A)** Linear Zeeman splitting of the $F = 1$ and $F = 2$ hyperfine manifolds. **(B)** The second order Zeeman shift gives an energy difference q between the states $|1, 0\rangle$ and $|1, \pm 1\rangle$, which is sketched in the co-rotating frame, here. Figure adapted from [19].

us access to all magnetic substates in both hyperfine manifolds. For the other measurements, like phase calibration or spectroscopy, where the low noise level is not needed, we use the imaging procedure with two separate frames.

To further optimize the pictures and to remove noise effects from optical fringes of the setup, we use an algorithm to construct an optimized reference picture for the absorption imaging [14, 32]. Here, we set up a library of ≈ 1000 reference pictures and then linearly combine them. The optimization process is performed on a region without atoms. Thus, the combination of many reference pictures decreases the photon shot noise by a factor of $\sqrt{2}$ and therefore leads to a precision of ± 25 atoms in each magnetic substate [14].

3.2.1 Projective measurements

In general a measurement of a quantum state is described by a projective measurement. This can be understood as a projection of the quantum state $|\psi\rangle$ onto the i -th eigenstate $\langle o_i|$ of the operator $\hat{O}$ with eigenvalue o_i

$$c_i = |\langle o_i|\psi\rangle|^2, \quad (3.1)$$

where c_i denotes the probability to measure o_i .

Measuring the atomic densities in each magnetic substate in our experiment via absorption imaging is a projective measurement. To understand the outcome of a measurement, we can use second quantization to express the corresponding measurement operators in each magnetic substate m_F by

$$\begin{aligned} \hat{N}_{+1} &= \hat{a}_{+1}^\dagger \hat{a}_{+1} = \frac{1}{3}\hat{N} + \frac{1}{2}\hat{S}_z + \frac{1}{4}\hat{Q}_{zz} \\ \hat{N}_0 &= \hat{a}_0^\dagger \hat{a}_0 = \frac{1}{3}\hat{N} - \frac{1}{2}\hat{Q}_{zz} \\ \hat{N}_{-1} &= \hat{a}_{-1}^\dagger \hat{a}_{-1} = \frac{1}{3}\hat{N} - \frac{1}{2}\hat{S}_z + \frac{1}{4}\hat{Q}_{zz}, \end{aligned} \quad (3.2)$$

where $\hat{N} = \sum_{m=-1}^1 \hat{N}_m$ is the operator for the total atom number. Thus, we can represent the number operators by the spin-1 operators. Having an ensemble of N particles, we will measure fluctuating atom numbers in each substate, corresponding to the underlying probability distribution of the quantum state and technical noise, e.g. from our setup. By repeating the same experimental procedure we can estimate the mean atom number $N_m = \langle \hat{N}_m \rangle$. Further, the representation from above allows us to directly extract for example the spin length in z -direction by calculating $N^z = N_{+1} - N_{-1} = S_z$. By applying a unitary transformations $\hat{U}$ we can rotate the state on the corresponding sphere and thus measure different observables. For example a $\pi/2$ rotation around $\hat{S}_y$ allows to measure the spin in x direction as the readout axis has been rotated from $\hat{S}_z$ to $\hat{S}_x$. The corresponding mapping onto the atom numbers can be determined using $N_m^f = \langle \hat{U}^\dagger \hat{N}_m \hat{U} \rangle$.

3.2.2 Imaging calibration

For the detection of squeezing and entanglement it is crucial to correctly determine the atom numbers from the measured absorption pictures. This is ensured by calibrating the imaging setup. To extract the atomic densities from the absorption image for each pixel n_{px} , with the size d_{px} , we use [14, 15]

$$n_{\text{px}} = \left[\frac{1}{c_{\text{cl}}} \ln \left(\frac{I_{\text{atom}}}{I_{\text{ref}}} \right) + c_{\text{I0}} \frac{I_{\text{atom}} - I_{\text{ref}}}{\tau_{\text{L}}} \right] \cdot \frac{d_{\text{px}}^2}{\sigma_0} . \quad (3.3)$$

Here, τ_{L} denotes the duration of the light pulse, σ_0 is the scattering cross section and I_{atom} and I_{ref} are the measured intensities of the atom picture and the reference picture, respectively. The first term is the commonly known Lambert-Beer law with the prefactor c_{cl}^{-1} . Further, the second term includes saturation effects, that are correctly taken into account by c_{I0} . The two calibration factors c_{cl} and c_{I0} are used to calibrate our readout. The idea behind the calibration is to prepare a polar state and evaluate the variance of the imbalance N^- which should yield the mean atom number, cf. equation (2.8). Thus, we start with the atoms in $|1, 0\rangle$ and clean the $F = 2$ hyperfine manifold and the states $|1, \pm 1\rangle$ by coupling them to the $F = 2$ manifold, i.e. $|1, \pm 1\rangle \leftrightarrow |2, \pm 1\rangle$, to prevent unwanted contributions from residual atoms due to off resonant spin-changing collisions. Then, we split the population by employing a microwave $\pi/2$ -pulse between the states $|1, 0\rangle$ and $|2, 0\rangle$. Applying a radio frequency $\pi/2$ -pulse creates an equal superposition between the states $|1, \pm 1\rangle$ and $|2, \pm 2\rangle$. To consider additional noise contributions we vary the mean atom number, by controlling the depth of the external potential during condensation. Then, we evaluate the variance of the measured atom number differences $N_F^- = N_{+F}^F - N_{-F}^F$ in the $F = 1$ and $F = 2$ manifold, respectively.

However, to further minimize correlated noise contributions, we split our evaluation region into two halves, left (L) and right (R). This reduces technical noise as this kind of noise is expected to be homogeneous over the atom cloud. Calculating the number difference in each half yields

$$\begin{aligned} N_{F=i}^- &= N_{\text{L}}^- - N_{\text{R}}^- \\ &= \left(N_i^{\text{L}, F=i} - N_{-i}^{\text{L}, F=i} \right) - \left(N_i^{\text{R}, F=i} - N_{-i}^{\text{R}, F=i} \right) , \quad i \in \{1, 2\} . \end{aligned} \quad (3.4)$$

For the calibration we exploit that the variance $\Delta^2 N_i^-$ for a coherent state is given by the mean atom number, i.e. $\langle N_{+F}^F + N_{-F}^F \rangle$. Thus, we expect a slope of one by plotting the variance as a function of the mean atom number. To calibrate the factors c_{cl} and c_{I0} we vary their values during the post analysis and calculate for each value pair the slope of the variance vs. the mean atom number. This evaluation yields a two dimensional parameter space, with the optimum value pair giving a slope of one. Further, we perform this measurement with different imaging intensities, as the second term in equation (3.3) dominates for higher intensities and the first term

3 Experimental control

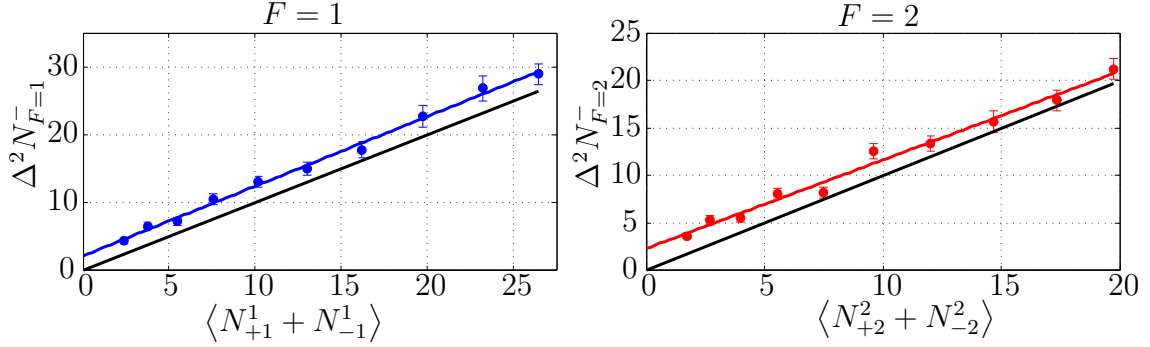


Figure 3.3: Imaging Calibration We calculate the variance of N_F^- and plot it as a function of the mean atom number $\langle N_i \rangle$ (dots). In black the theoretical prediction, i.e. a slope of 1 without offset, is shown. For our calibration measurement we find a slope of $a_1 = 1.030 \pm 0.027$ for $F = 1$ and $a_2 = 0.94 \pm 0.04$ for $F = 2$. The offsets are found to be 2105 ± 414 for $F = 1$ and 2269 ± 469 for $F = 2$. Here, the photon shot noise of 1596 and 1309 for $F = 1$ and $F = 2$, respectively, is included.

becomes more important for lower intensities. Thus the parameter space becomes three dimensional. For the squeezing measurements presented in this thesis, we did a calibration, whose result is summarized in figure 3.3. In the figure the intensity is comparable to the intensity used for our measurements.

3.3 RF and MW manipulations

Having outlined the underlying experimental platform, we discuss manipulations on the system, in the following. As summarized in figure 3.2 we deal with two energy scales. While the energy spacing between the two hyperfine manifolds is on the order of ≈ 6.7 GHz, the energy between the substates inside a manifold is only ≈ 700 kHz. Thus, in our setup we use radiofrequency (rf)- and microwave (mw) fields, to control spin rotations (rf), transfer atoms between the two hyperfine manifolds $F = 1, 2$ (mw) and imprint phases onto the atoms (mw). This section discusses the setup and the underlying theoretical description.

3.3.1 RF pulses

We use our rf-setup to induce spin rotation, as indicated before. The rf-setup consists of two coils, that are controlled by an arbitrary waveform generator (Agilent 33522A), which has two separately addressable output ports. This allows to individually control the coils. Generating an rf-signal with the waveform generator, each of the two coils creates a linearly polarized magnetic field $\vec{B}_{\text{rf},i} = B_{\text{rf},i} \cos(\omega_{\text{rf},i} + \phi_{\text{rf},i}) \vec{e}_i$, with $i \in \{1, 2\}$ and $\vec{e}_1 \parallel x$ and $\vec{e}_2 \parallel y$, respectively. Here, $\omega_{\text{rf},i}$ and $\phi_{\text{rf},i}$ denote the frequency and the phase of the field created by coil i . The coils are placed such that the resulting rf-field is perpendicular to the magnetic offset field (x and y direction), cf. figure 3.1 A. Thus the time dependent Hamiltonian for the $F = 1$ manifold is

given by

$$\mathcal{H}_{\text{rf}}(t) = 2\hbar\Omega_{\text{rf},0} \cos(\omega_{\text{rf}}t + \phi_{\text{rf}})\hat{S}_y, \quad (3.5)$$

with the resonant Rabi frequency $\Omega_{\text{rf},0} = |g_{1,1}|B_{\text{rf}}/2$ and $g_{1,1}$ the linear g-factor for the $F = 1$ manifold. To further investigate the influence of an rf field onto the system, we transform it into the rotating frame of the rf pulse. For a resonant rf-pulse $\omega_{\text{rf}} = 2\pi \cdot p_{\text{B}}$, cf. figure 3.2 A, we get

$$\begin{aligned} \mathcal{H}_{\text{rf}}^{\text{rot}} &= e^{-i\omega_{\text{rf}}t\hat{S}_z} \mathcal{H}_{\text{rf}} e^{i\omega_{\text{rf}}t\hat{S}_z} \\ &= \hbar\Omega_{\text{rf},0} \left[\cos(\phi_{\text{rf}})\hat{S}_y - \sin(\phi_{\text{rf}})\hat{S}_x \right]. \end{aligned} \quad (3.6)$$

Here, we used the rotating wave approximation and neglected all terms with a frequency $2\omega_{\text{rf}}$. Subsequently, the last line shows, that an rf-pulse employs a Rabi oscillation with the frequency $\Omega_{\text{rf},0}$, where the rotation axis is determined by the externally applied rf-phase ϕ_{rf} .

Applying an rf pulse with a detuning $\delta = \omega_0 - \omega_{\text{rf}}$, leads to a precession of the state around the z axis. Then, the rotation axis is rotated out of the plane spanned by $\hat{S}_x$ and $\hat{S}_y$ and is described by $-\hbar\delta\hat{S}_z$, which yields the following Hamiltonian

$$\mathcal{H}_{\text{B}}^{\text{rot}} = \hbar\Omega_{\text{rf},0} \left[\cos(\phi_{\text{rf}})\hat{S}_y - \sin(\phi_{\text{rf}})\hat{S}_x \right] - \hbar\delta\hat{S}_z. \quad (3.7)$$

It is to note, that the rotation axis is defined by the phase difference of the rf-pulse and the Larmor phase. However, initially it is not possible to define the Larmor phase ($\phi_{+1} - \phi_{-1}$) as after condensation all atoms occupy the state $|1, -1\rangle$. Thus, the first rf-pulse acts as reference and we define it to be a rotation around the $\hat{S}_y$ axis. The same arguments hold for a preparation of the coherent state in $|1, 0\rangle$. The above treatment can be directly applied to $F = 2$, by replacing the spin-1 operators with the corresponding spin-2 operators.

Further, the resonant frequency depends on the g-factor, which has a similar absolute value in both hyperfine manifolds ($|g_{1,1}| \approx |g_{1,2}|$). For a magnetic field of 1.5 G this yields a difference of $\approx 2\pi \cdot 4.4$ kHz. We use this circumstance to apply a simultaneous rf pulse in both hyperfine manifolds. With a detuning of $\approx 2\pi \cdot 2.2$ kHz to each resonant frequency $\approx 2\pi \cdot 1$ MHz, this detuning becomes negligible and in both manifolds a simultaneous spin rotations is employed.

3.3.2 MW pulses

While the rf-pulses induce spin-rotations in one manifold, we use mw-fields to transfer atoms between the two manifolds. To describe the mw transition it is useful to reduce the description to a two-level system. Then, we can define the mw transition

3 Experimental control

operators using second quantization, which yields

$$\hat{C}_x^{m_1, m_2} = \frac{1}{2} \hat{a}_{1, m_1}^\dagger \hat{a}_{2, m_2} + \text{h.c.} \quad \hat{C}_y^{m_1, m_2} = -\frac{1}{2} i \hat{a}_{1, m_1}^\dagger \hat{a}_{2, m_2} + \text{h.c.} \quad (3.8)$$

Here, $\hat{a}_{F, m_F}^{(\dagger)}$ denotes the annihilation (creation) operator in the state $|F, m_F\rangle$ and h.c. the hermitian conjugate. The index x, y indicates the rotation axis of the pulse. For example, in a two level system, i.e. on the Bloch-sphere, the unitary operator $e^{-i\phi\hat{C}_x}$ corresponds to a rotation around the x axis. Further, we have to take the dipole selection rules into account, to determine allowed transitions. The selection rules give us only couplings with $\Delta m_F = m_1 - m_2 = \{0, \pm 1\}$.

Besides using mw-transitions for state preparation, we use mw-dressing to initiate spin-mixing dynamics. Our mw-setup consists of two commercial mw generators (HP 8673D and HP 83620A). While we usually use the first generator for state preparation and readout, we employ dressing with the second generator. In contrast to the first mw generator, the second mw generator (HP 83620A) has an internal control circuit, which allows a higher mw power stability compared with the first generator. Here, we couple a part of the signal after the amplifier into the internal control circuit, such that the mw power is actively stabilized [33]. With this the setup is suitable for spin mixing dynamics, where a stable mw power is needed, which is the topic of the next paragraph.

3.3.3 Detuning and spin-mixing

The relevant energy scale for spin-mixing dynamics is the energy difference of the state $|1, 0\rangle$ compared with the states $|1, \pm 1\rangle$ due to the second order Zeeman shift, see equation (2.18). Thus, by controlling this energy difference, we are able to regulate the spin-mixing dynamics. In our experiment, we use mw dressing to shift the energy of $|1, 0\rangle$ via the ac-Stark shift by off-resonantly coupling to $F = 2$ [34]. The resulting effective detuning is given by

$$q_{\text{eff}} = q_B + q_{\text{mw}} \quad , \quad \text{with} \quad q_{\text{mw}} = \frac{\Omega_{\text{mw}}^2}{4\delta_{\text{mw}}} \quad (3.9)$$

Here, Ω_{mw} denotes the Rabi frequency, δ_{mw} the detuning of the mw and $q_B \approx 2\pi \cdot 150 \text{ Hz}$ the second order Zeeman shift, which depends on the external field. Consequently, by changing the detuning of the mw field, we have the ability to control the spin-mixing process.

To determine the correct detuning, we employ a spectroscopy-like sequence before each measurement.

3.3.4 Spectroscopy sequence

Tuning spin-mixing processes into resonance allows to populate excited spatial modes. However, due to interactions between the atoms these do not correspond to the

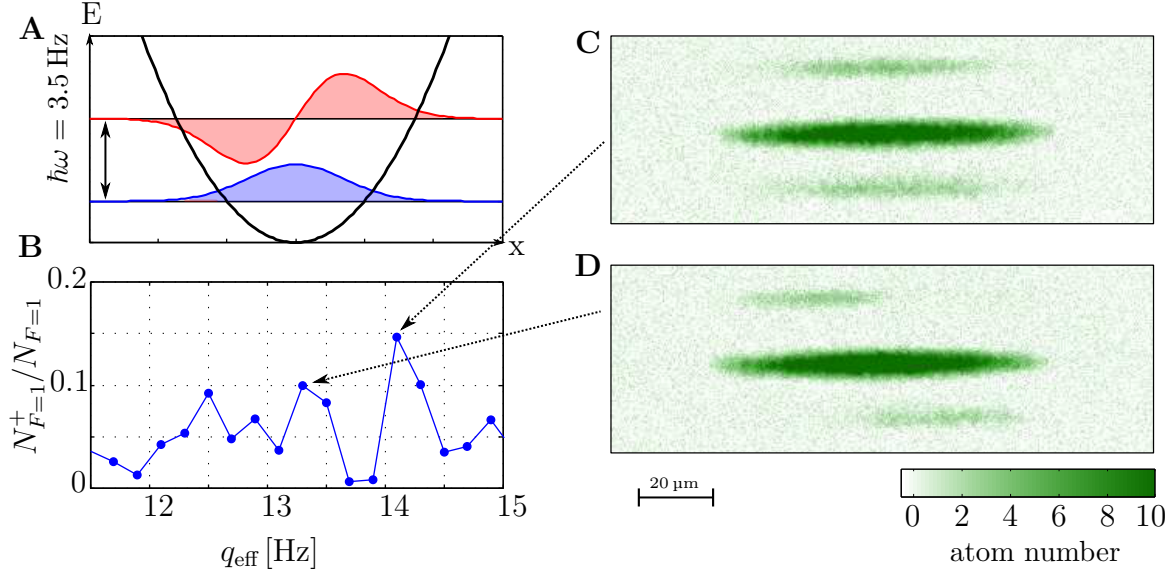


Figure 3.4: Spin changing collisions spectroscopy: (A) Schematics of the effective potential. For the two energetically lowest modes, the potential is harmonic with an energy spacing of $\approx 2\pi \cdot 3.5$ Hz. (B) By changing the detuning of the mw pulse, we change q_{eff} , which allows to spontaneously populate different spatial modes. (C) and (D) From large to small q , we first excite the ground mode (C) and then the first excited mode (D), with its asymmetric behavior.

spatial modes of the trapping potential, which would cause an energy spacing of $\Delta E/\hbar \approx 2\pi \cdot 50$ Hz in the harmonic XdT, cf. section 3.1. Furthermore, the interactions lead to an effective potential on top of the condensate [14]. For the two energetically lowest modes this corresponds to a harmonic potential with an energy spacing of $\Delta E/\hbar \approx 2\pi \cdot 3.5$ Hz. The harmonic potential yields the well-known spatial structure of wavefunctions as found as well for the quantum harmonic oscillator. The spatial wavefunction for the two energetically lowest modes are depicted in figure 3.4 A. We see, that the first excited mode has a gaussian profile and the second excited mode an “up/down” structure between the left and right half and a zero crossing in the middle.

For energetically higher modes, i.e. the third mode and upwards, the potential becomes box-like, which yields the corresponding sinusoidal eigenfunctions. A full treatment and discussion of the effective potential is given in [14]. However, for the detection of entanglement presented in this thesis only the two energetically lowest modes are important.

To identify the spin-mixing resonance we scan q_{eff} over the value range in which the $m_F = \pm 1$ states are spontaneously populated, see figure 3.4. For this we start with all atoms in $|1, 0\rangle$ and employ mw-dressing at a magnetic field of 1.46 G for 800 ms. Measuring the sidemode population $N^+ = N_{+1} + N_{-1}$ spatially resolved directly enables us to determine the corresponding spatial mode in resonance.

This sequence leads to the measurement result shown in figure 3.4 B. Additionally,

3 Experimental control

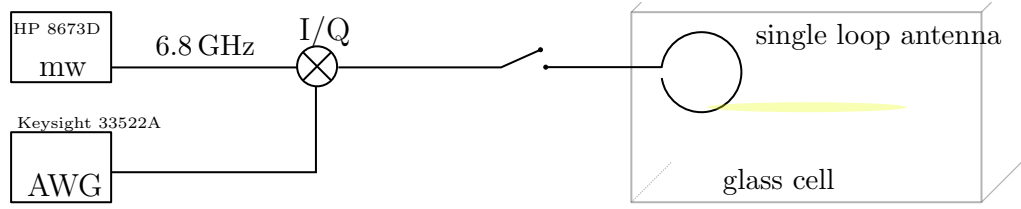


Figure 3.5: Microwave setup: To create mw signal we mix a radiofrequency signal from an AWG with a constant mw signal from a mw generator using an I/Q mixer. The AWG allows to control the frequency and phase of a pulse with high stability.

we indicate the imprinted spatial structures onto the states $|1, \pm 1\rangle$ in figure 3.4 C and D, respectively. For our measurements we want to bring the two lowest modes into resonance. Thus, we select a detuning in-between both. This corresponds to a point ≈ 2 Hz from the right edge of the spectroscopy result, cf. figure 3.4 B.

3.3.5 Microwave control using arbitrary waveform generators

In the previous paragraph we focused on the control of spin-mixing dynamics, which are employed in the $F = 1$ manifold. To drive transition between the two hyperfine manifolds $F = 1$ and $F = 2$ we also use mw-fields. In this setup we use a mw generator, which runs on a fixed frequency of 6.8 GHz and is mixed with a radio frequency signal using an I/Q mixer. We use a programmable rf source of which we can control the frequency, phase and amplitude with high precision. In an earlier setup for this a direct digital synthesizer (DDS) was used as an rf source. However, with new arbitrary waveform generators (AWG) and a newly programmed communication and calculation we could further improve this setup. A sketch of this setup is shown in figure 3.5. An AWG gives us full control over the output signal and thus allows us to create all sorts of signals and pulse sequences with high precision [18]. However, due to memory and computational constraints a direct definition of the sequence is usually not possible. In the following chapter we describe the methods to obtain the high level of control necessary for an ultracold atom experiment. In our setup we use type Keysight AWGs that are controlled via MATLAB.

The working principle of AWGs is, that they apply a given voltage at each time step. A time steps corresponds to a sample point in the internal memory, in which the voltage value is stored. Thus, by successively calling each sample a signal is created. The resolution and the frequency of the generated signal depend on the duration of each sample τ_{Sample} , which is given by the sample rate $f_{\text{Sample}} = 1/\tau_{\text{Sample}}$. The Keysight AWGs that we use in our experiment can operate up to a maximal sample rate of 1 GSa/s depending on the model. This results in a duration of ≈ 1 ns of each sample. Nyquist's theorem states that a signal containing a maximum frequency f_{max} needs at least two samples per period. Thus $f < 500$ MHz are possible. However, to obtain a more precise and stable signal we usually use this generator for frequencies around 100 MHz. By applying a sinusoidal signal for 60 μs , which is a typical time scale for pulses in the experiment, we would need 6×10^4

samples if operating at the highest possible sample rate. Employing more pulses of this type, linearly increases the number of samples and eventually the internal memory of the device cannot store more samples. Limited by Nyquist's theorem and the internal memory we want to find a compromise between long sequences and occurring sidebands due to low sample rates. Here, we make use of the function of the AWGs, that allows to repeat a short segment up to 10^6 times before jumping to the next segment. Using this, we need to program only one period of a periodical signal into the AWG, which is then repeated. However, during the calculation several aspects have to be considered. These are explained in a detailed technical description in the appendix B.

3.3.6 Capabilities of arbitrary waveform generator setup

We compared the new setup, i.e. the new AWG with its new code, to the old setup with a DDS, that was used in previous experiments. To compare both setups we implemented a Ramsey sequence. We prepared the atoms in the state $|F, m_F\rangle = |1, 0\rangle$ and applied a $\pi/2$ pulse to couple the initial state with the state $|2, 0\rangle$. Then, we hold the atoms for a fixed time of $\approx 100\ \mu\text{s}$ and coupled the states with another $\pi/2$ pulse with a fixed phase. We chose this phase such that we create an equal superposition between the states $|1, 0\rangle$ and $|2, 0\rangle$ after the second pulse. Turning to a two-level description of the states $|1, 0\rangle$ and $|2, 0\rangle$, we represent the system on the Bloch-sphere, with $|1, 0\rangle$ being on the north pole and $|2, 0\rangle$ on the south pole. The first mw pulse rotates the state after around the y -axis onto the equator, where the free evolution lets the state rotate on the equator around the z -axis. The phase of the second mw pulse is chosen such that the rotation axis is aligned with the state after $\approx 100\ \mu\text{s}$. Thus, if the relative phase between the two pulses is not stable, the second rotation rotates the state away from the equator and will create an imbalance between the states $|1, 0\rangle$ and $|2, 0\rangle$. This can be measured by evaluating the atom number difference

$$N^- = \frac{N_0^{F=1} - N_0^{F=2}}{N_0^{F=1} + N_0^{F=2}} . \quad (3.10)$$

Here, $N_{m_F}^F$ denotes the measured atom numbers in the state $|F, m_F\rangle$. We compare the phase stability of both setups after switching frequencies. Switching the frequency to an off resonant frequency, i.e. in this case the frequency between the states $|1, 1\rangle \leftrightarrow |2, 2\rangle$, during the hold time and back before the second pulse allows us to characterize the switching behavior of the DDS. For the AWG we expect, that it can change the pulse on the level of a single sample. As a reference we also measured the number difference N^- without applying the frequency jump during the hold time. The measurement results of these two sequences are shown in figure 3.6. To benchmark the new setup we applied the same sequences to the old setup, where the DDS generator was used for the pulses.

The results show, that the setup with the arbitrary waveform generator has a

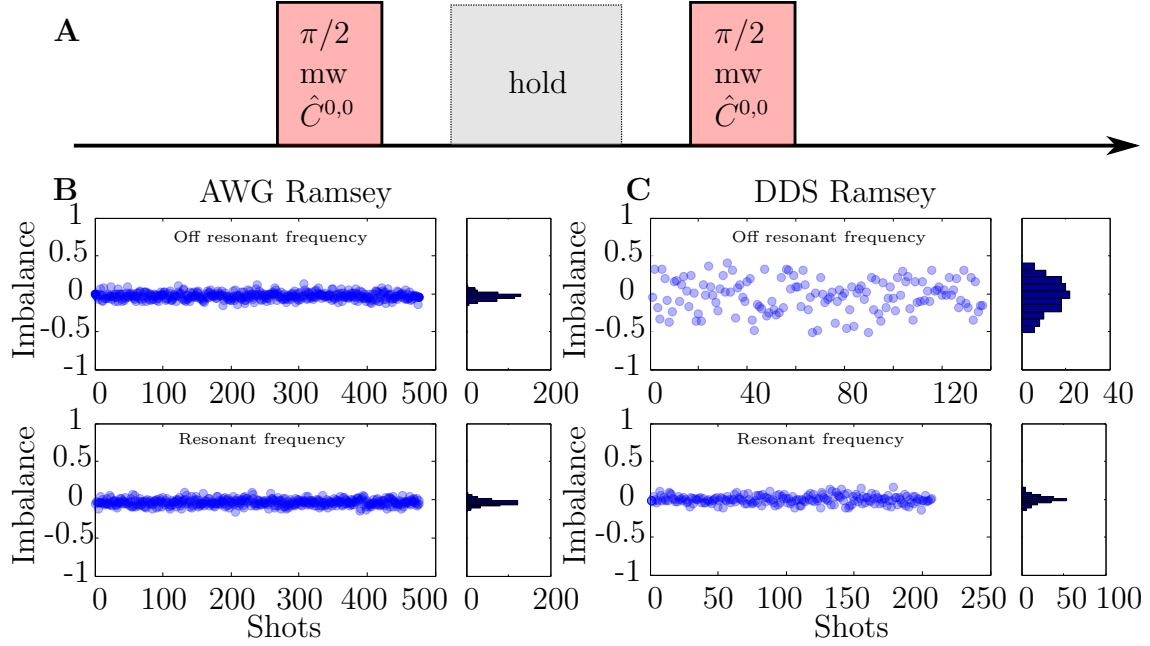


Figure 3.6: Comparison of new mw-setup with old setup: We compare the old and the new microwave setup by applying a Ramsey sequence. (A) Schematics of the probing sequence. During the hold time, we switch the frequency of the AWG or DDS. (B) Normalized imbalance for the new setup with the arbitrary waveform generator. During the hold time we switch to an off resonant frequency (upper row) and for comparison we do not switch the frequency (lower row). We find a variance of $\sigma^2 = 35.7 \pm 2.7$ for the switching to the off resonant frequency and $\sigma^2 = 32.4 \pm 2.2$ for staying at the same frequency. Thus, the imbalance scatters in a small regime around 0 in both cases. (C) We perform the same measurement to the old setup. Especially in the case, where we switch to an off resonant frequency we extract a higher standard deviation of $\sigma^2 = 704 \pm 77$, while for the fixed frequency we get a standard deviation of $\sigma^2 = 124 \pm 14$.

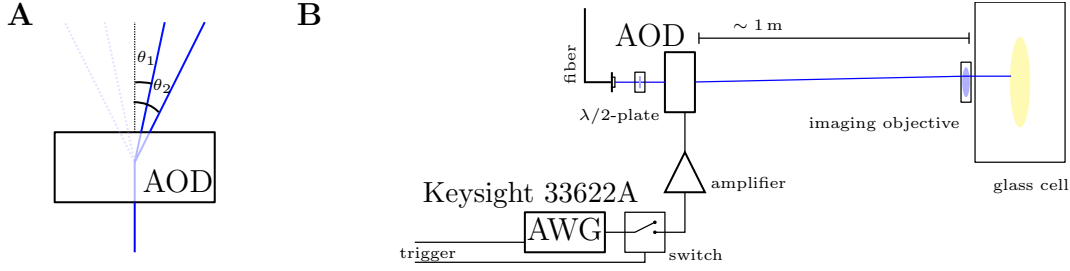


Figure 3.7: Box setup (A) Schematics of the working principle of an AOD. Feeding a radio frequency signal with two frequencies into an AOD allows to split a blue detuned laser beam and create repulsive barriers. (B) Setup of AODs with arbitrary waveform generator. Having two independent output ports at the AWG allows to drive two AODs and thus control the horizontal and vertical position of the barrier.

better phase stability compared with the setup using the DDS. While the two measurement types with the AWG have a comparable spread in the imbalance, i.e. the standard deviation is on the same order, the measurement with the DDS shows a much wider spread for both measurements. Furthermore, when switching to an off-resonant frequency the standard deviation rises up to $\sigma^2 = 704 \pm 77$, which is 20 times more than the AWG setup. This indicates a higher phase stability of the AWG setup compared with the DDS setup.

3.4 Box-like potential

Further, we used the AWG in combination with an acousto-optic deflector (AOD) to create a box-like potential. In an AOD optical light is deflected by an angle θ , depending on the frequency f of the input signal. Thus, by applying two different frequencies f_1 and f_2 we get two output angles θ_1 and θ_2 . Setting up the AOD as summarized in figure 3.7 and shining in a blue-detuned laser, creates a repulsive barrier in the atom cloud. The position of the barrier depends on the angle θ_i and the distance between the AOD and the atoms. The working principle of an AOD is sketched in figure 3.7 A. Further, the control of the AODs using an AWG is shown in figure 3.7 B. Using the two available output ports of the AWG, we individually control two AODs. These AODs are placed orthogonal to each other and allow to deflect the laser beam in a vertical and horizontal direction. Additionally, we use a pellicle to couple the AOD beam path into the imaging beam path. This should be considered for the stability of the setup due to mechanical vibrations.

To elucidate the resulting effect on the atoms, we show atom densities of different setups, consisting of 1-3 boxes in figure 3.8. The atom density pictures clearly show the steep slope at the edges and the homogeneous density in-between. With this new setup we can create potentials beyond a harmonic trap and thus investigate physical phenomena in a new situation where edge effects of a harmonic trap become less dominant. Further, we are able to increase our number of experimental realizations as we have up to three “experiments”.

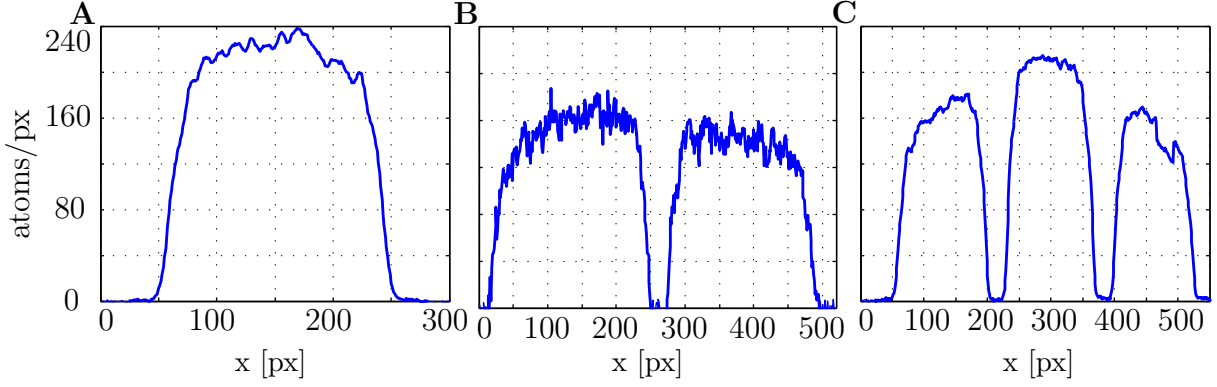


Figure 3.8: Box-like potential To show the capabilities of the new setup we prepared three different cases. (A) Overlapping two frequencies and forwarding them into the AOD, creates a single box like potential. We show the averaged profile over ≈ 100 realizations. In this setting we employed spin-mixing dynamics. (B) With three different frequencies we are able to create two boxes. (C) Here we show a total of three boxes with the averaged density profile.

4 Simultaneous readout of conjugate observables

To detect squeezing, we use a spatially resolved extraction of the two non-commuting observables S_x and Q_{yz} [35].

The idea is to use an initially empty hyperfine manifold to enlarge the dimension of the accessible Hilbert Space. Applying unitary transformations in each manifold allows to measure even non-commuting observables in a single shot. However, this measurement introduces additional fluctuations and therefore one cannot measure two conjugate observables with infinite precision guaranteeing agreement with Heisenberg’s uncertainty. In the following chapter we introduce the readout sequence and the corresponding calibration measurements to obtain a low-noise measurement result.

4.1 Experimental sequence and readout scheme

We start with explaining the employed experimental sequence with emphasis on the simultaneous readout technique of two observables. We detect the two observables S_x and Q_{yz} by mapping them onto the measurable number differences N_F^- in the $F = 1$ and $F = 2$ manifold, respectively. The starting point for our experiments is all atoms occupying the state $|F, m_F\rangle = |1, 0\rangle$. As after condensation all atoms are in the state $|1, -1\rangle$, cf. section 3.1, we implement a transfer to the state $|1, 0\rangle$ by coupling the states $|1, -1\rangle \leftrightarrow |2, 0\rangle$ and $|2, 0\rangle \leftrightarrow |1, 0\rangle$ with two consecutive mw π -pulses. Applying a strong magnetic field gradient after the transfer cleans any residual atoms in $|1, -1\rangle$. Then, we wait another 100ms to let the magnetic field stabilize, before we begin our experiment. During these 100ms spontaneous spin-changing collisions might have occurred which could populate the states $|1, \pm 1\rangle$. Or residual atoms are left in $|2, 0\rangle$ after the transferring process. To clean these levels, we apply an optical cleaning sequence. First, we employ an image pulse to

the $F = 2$ hyperfine manifold. Second, we couple the states $|1, -1\rangle \leftrightarrow |2, -1\rangle$ and $|1, 1\rangle \leftrightarrow |2, 1\rangle$ with microwave π -pulses. This transfers any residual population from $|1, \pm 1\rangle$ to $|2, \pm 1\rangle$, where a second image pulse is applied. These image pulses push residual atoms out of focus and we are left with only the state $|1, 0\rangle$ populated.

After this additional cleaning, we initiate spin-mixing dynamics by mw dressing. Here, we employ a far off-resonant mw-pulse to couple the states $|1, 0\rangle \leftrightarrow |2, 0\rangle$. The corresponding detuning has been extracted using a spectroscopy sequence, cf. section 3.3.4. This coupling changes the energy of the state $|1, 0\rangle$ with respect to the states $|1, \pm 1\rangle$ and thus initiates the dynamics. In figure 4.1 we present a schematic overview of the dynamics and the readout sequence. After a fixed evolution time τ_{Evo} , where we leave the mw-dressing on, we apply three mw $\pi/2$ -pulses coupling the states $|1, k\rangle \leftrightarrow |2, k\rangle$ with $k \in \{0, \pm 1\}$. This splits the population in half and transfers one half to the $F = 2$ manifold. This splitting increases our accessible Hilbert space. Coupling the $m_F = 0$ states introduces a Spinor phase ϕ_S . Additionally, the mw pulses coupling the sidemodes act as a reference for the Larmor phase. We choose their phases as $\phi_{m_F} = \phi_{+1} = -\phi_{-1}$. These mw pulses are done using the mw-AWG setup to obtain a high phase stability and reproducibility of the pulses. The sequences of the pulses is indicated in figure 4.1. A final simultaneous rf $\pi/2$ -pulse in $F = 1$ and $F = 2$, with a detuning of $2\pi \cdot 2.2$ kHz maps the observables S_x and Q_{yz} onto the measurable number differences N_F^- in the $F = 1$ and $F = 2$ manifold, respectively.

Having explained the sequence, we now determine the mapping of the observables onto the sidemode populations. For this, we look at the unitary operator for the readout sequence $\hat{U}$, i.e. after the evolution time. The operator is given as

$$\hat{U} = e^{-i\frac{\pi}{2}\hat{S}_x^{F=2}} \cdot e^{-i\frac{\pi}{2}\hat{S}_x^{F=1}} \cdot e^{-i\frac{\pi}{2}\hat{C}_y^{+1+1}} \cdot e^{-i\frac{\pi}{2}\hat{C}_y^{-1-1}} \cdot e^{-i\frac{\pi}{2}\hat{C}_x^{00}}. \quad (4.1)$$

Here, the operators $\hat{C}_k^{m_1, m_2}$, with $k \in \{x, y, z\}$ denote the mw-pulses between the two magnetic substates m_1, m_2 and $\hat{S}_k^{F=i}$ the rf-pulse in the hyperfine manifold $F = i$, with $i \in \{1, 2\}$. The spin operator $\hat{S}^{F=2}$ is given in appendix A.2. The operator $\hat{U}$, cf. equation (4.1), then allows us to calculate the mapping of the observables onto the atom number using $\tilde{N}_m^F = \langle \hat{U}^\dagger \hat{N}_m^F \hat{U} \rangle$, where N_m^F denotes the population in the magnetic substate m_F and the hyperfine manifold $F = 1, 2$. Using the relation found in equation 3.2 and the unitary operator in equation (4.1) we find the following expression for the observables

$$\begin{aligned} S_x(x) &= N_{+1}^{F=1}(x) - N_{-1}^{F=1}(x) \\ Q_{yz}(x) &= \frac{2}{\sqrt{3}} (N_{+2}^{F=2}(x) - N_{-2}^{F=2}(x)) , \end{aligned} \quad (4.2)$$

with $N_{m_F}^F$ the population of the state $|F, m_F\rangle$.

A possible external influence on the scheme are the phases of the three mw split pulses. These are calibrated once before each longer measurement.

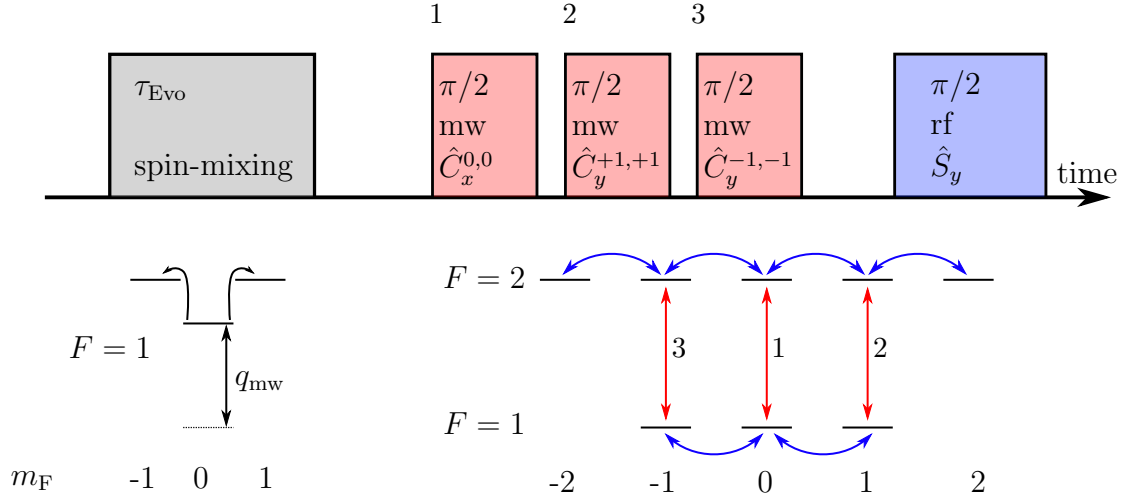


Figure 4.1: Readout sequence: (**Upper row**) After a fixed evolution time τ_{Evo} of spin-mixing dynamics, we split the population into two halves by transferring half the population to $F = 2$ with three mw $\pi/2$ -pulses. A simultaneous rf $\pi/2$ -pulse maps the observables S_x and Q_{yz} onto the magnetic substates. (**Lower row**) Visualization of the mw dressing (left) and the mw (red) and rf (blue) manipulations (right) in a level scheme.

4.2 Phase calibration

To calibrate the phases of the three mw pulses, we change the readout scheme a bit. We calibrate for the Spinor phase $\phi_S = 0$ and add a Spinor phase shift of $\delta\phi_S = 90^\circ$ afterwards to readout the orthogonal observable. In the calibration we scan the two dimensional parameter space of the Larmor and Spinor phase. For this we imprint either a Larmor or Spinor phase prior to the readout sequence. Depending on the variable phase imprint and the fixed phase setting of the mw split pulses we want to find the phases such that we detect the same observable in $F = 1$ and $F = 2$. The pulse sequence is sketched in figure 4.2 A. As before, we start with all atoms in the state $|1, 0\rangle$. Then, an rf $\pi/4$ -pulse is applied, that changes the initial polar state into a so-called tilted polar state. This pulse defines the Larmor phase because it populates the side-modes $|1, \pm 1\rangle$. In the next step, two consecutive mw π -pulses between the states $|1, 0\rangle \leftrightarrow |2, 0\rangle$ imprint a variable Spinor phase to the state $|1, 0\rangle$. Transferring the atoms from $|1, 0\rangle$ to $|2, 0\rangle$ and back allows to imprint a Spinor phase by having a phase difference between the two mw pulses. After these two phase imprints, three mw $\pi/2$ split pulses with fixed phases as well as an rf $\pi/2$ -pulse are applied. These last pulses are similar to the previously explained readout scheme. The phases of the split pulses are chosen as $\phi_{m_F} = \phi_0$ and $\phi_{+1} = -\phi_{-1}$ and kept constant over the calibration. To extract the correct phases, yielding the same readout in $F = 1$ and $F = 2$, we vary the Larmor phase, imprinted by the rf $\pi/4$ -pulse and keep the other phases fixed. In a second setting, i.e. after we scanned the Larmor parameter space, we vary the Spinor phase introduced by the two mw

π -pulses between the states $|1, 0\rangle$ and $|2, 0\rangle$ before the readout sequence. In this second setting, we keep the Larmor phase imprint fixed. Varying the Spinor phase rotates the state on the equator of the spin-nematic sphere (cf. figure 2.2) between a fully elongated spin in x -direction and the eigenstate of the operator $\hat{Q}_{yz}$, such that $\langle S_x \rangle = 0$. A variation in the Larmor phase changes the state between an elongated spin into x and y direction, i.e. on the equatorial plane of the spin sphere. From the measurement we extract the imbalance $N_F^- = N_{+F}^F - N_{-F}^F$ for $F = 1, 2$. This corresponds to a measurement of $\langle S_x \rangle = \langle N_F^- \rangle$. For each setting (Spinor and Larmor phase imprint) we find a phase such that the state is aligned with the readout axis and we get a full signal. This corresponds to a maximal imbalance in the measured atom number difference. By changing the phase imprint, we slowly turn the state orthogonal to the readout axis, resulting in no measurable imbalance, i.e. $N^- = 0$. Thus, calculating the atom number differences in both hyperfine manifolds yields a sinusoidal fringe in each manifold. For each parameter space (Larmor and Spinor phase) two exemplary fringes are shown in figure 4.2.

Here, we find that the phases of the mw pulses are not correctly calibrated. This leads to a relative phase difference between the two fringes for each scanned parameter. To account for this phase difference, the phases of the three split pulses have to be adapted. Calculating the phase difference via

$$\delta\phi_{S/L} = \phi_{F=2} - \phi_{F=1} , \quad (4.3)$$

where $\phi_{F=i}$ denotes the phase offset of a sine with positive amplitude for both manifolds and S/L the scanned parameter space (Spinor/Larmor phase), allows to directly determine the phase adjustment. For the Spinor phase correction (i.e. Larmor phase is constant) we get

$$\phi_S^{\text{new}} = \phi_S^{\text{old}} - \delta\phi_S \quad (4.4)$$

and for the Larmor phase correction

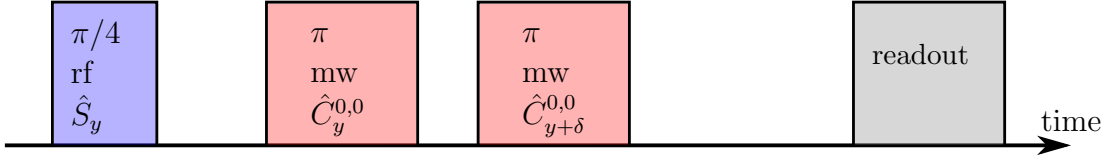
$$\phi_L^{\text{new}} = \phi_L^{\text{old}} + \delta\phi_L . \quad (4.5)$$

Here “old” and “new” denote the phases of the mw split pulses used in the calibration and after the correction, respectively. For the experiment the Spinor phase is then shifted by 90° to read out the orthogonal quadrupole observable in $F = 2$.

4.3 Direct sampling of the phase space

This calibrated new readout scheme allows us to directly sample from the phase space distribution of S_x and Q_{yz} . With additional access to the spatial degree of freedom we are able to evaluate different spatial regions independently. To see this, we tune the spin-mixing dynamics into resonance with the third excited mode of the external potential. We let the system evolve for 800 ms before applying the readout scheme. The ongoing dynamics can be clearly seen in the absorption image, shown in

A



B

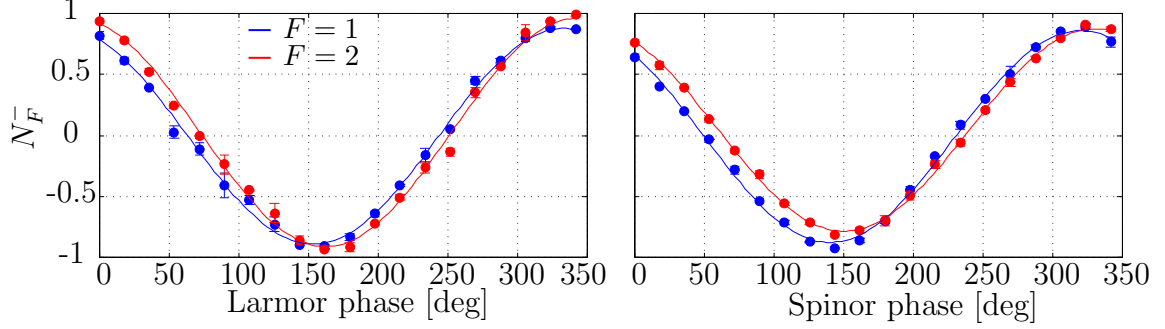


Figure 4.2: Phase calibration: (A) We apply phase imprints during the rf pulse and the two mw pulses, which affect the Larmor and Spinor phase, respectively. While varying one of them we keep the other constant. After the phase imprint, we apply our readout sequence. (B) Measurement result for a phase calibration with constant Spinor imprint (left) and constant Larmor phase (right), respectively. In both cases, the fringes do not lay on top of each other, thus we have to adjust the Spinor and Larmor phase.

figure 4.3. Each of these experimental realizations yields a sample of the phase space distribution. Now we split the cloud horizontally into four equally sized regions and evaluate them separately. This yields four different phase space distributions, which are shown in figure 4.4. Each of these subsystems has evolved beyond the Gaussian regime along the trajectories mentioned in section 2.2.3. Figure 4.4 highlights the capabilities of the new readout scheme and the evolution of the state beyond a Gaussian distribution.

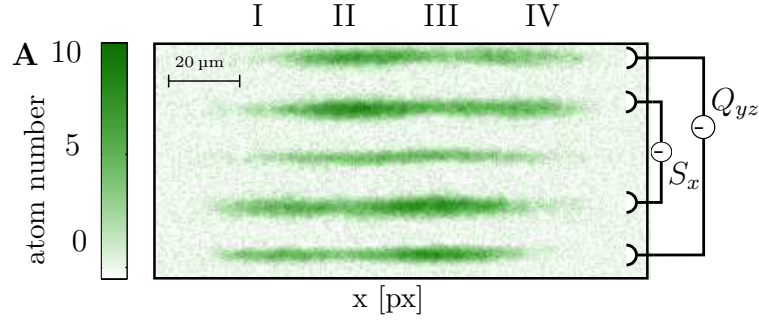


Figure 4.3: Absorption picture: We tuned spin-mixing processes into resonance with the third excited mode. The picture is a single experimental realization and shows the ongoing dynamics in the system. On the right side the extraction of the observables S_x and Q_{yz} from the atom numbers is depicted.

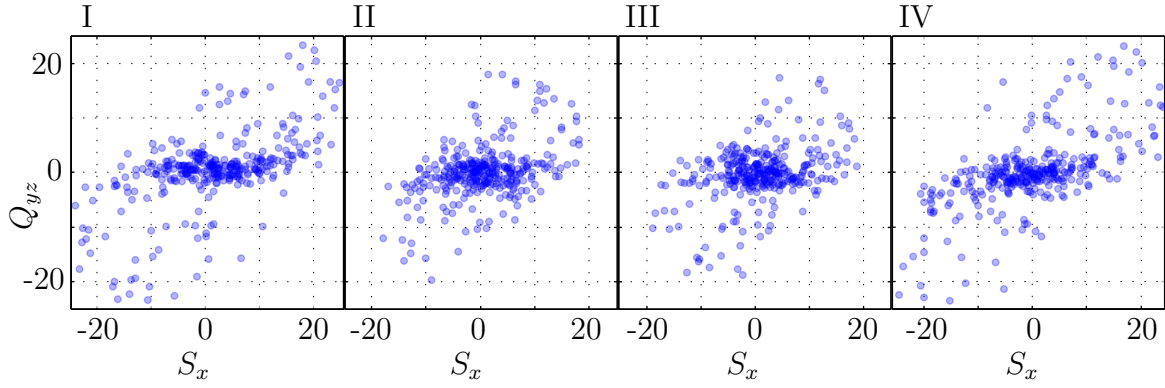


Figure 4.4: Phase-space: Each experimental realization corresponds to a direct sampling from the phase space distribution. Evaluating different spatial subsystems corresponds to a partial trace and yields a phase space distribution for each subsystem, depicted as I-IV. Each subsystem follows the trajectories shown in figure 2.3. For a better visualization we normalized the data to $\sqrt{N}$.

5 Detection of Squeezing and Entanglement

In this chapter, we discuss our strategy to detect squeezing by analyzing the variance of the measured atom numbers and motivate our entanglement witness. As we discussed earlier the readout sequence is very sensitive. To investigate its capabilities and understand our evaluation method we use truncated Wigner simulations.

5.1 Variance analysis

Having small atom numbers in the sidemodes after spin-mixing dynamics, makes it experimentally hard to directly determine the atom number. However, the spin-mixing process changes the second moment of the probability distribution of an initial polar state [14]. Employing a homodyne-like measurement allows to detect the small sidemode population due to the spin-mixing process. In chapter 4.1 we introduced the relation between the observables S_x and Q_{yz} and how we can extract them from the atom numbers, cf. equation (4.2).

As the squeezing axis might not be aligned with neither of the observables it is convenient to define a new observable

$$F(\phi) = \cos(\phi)S_x + \sin(\phi)Q_{yz} , \quad (5.1)$$

that is a linear combination of the two observables S_x and Q_{yz} . This observable allows to analyze any direction, characterized by the correlation angle ϕ , of the phase space spanned by S_x and Q_{yz} . Thus, evaluating the variance of the distribution of $F(\phi)$ allows us to detect squeezing. As we have seen during the discussion of spin squeezing, cf. section 2.3, a state is squeezed if the variance is reduced below that of a coherent state. In our case this is the polar state. To have a measure of squeezing, we normalize the variance of $F(\phi)$ such that we get for a coherent state (CS)

$$\frac{\Delta^2 F(\phi)_{\text{CS}}}{\langle N(\phi) \rangle_{\text{CS}}} = 1 , \quad (5.2)$$

where

$$\langle N(\phi) \rangle_{\text{CS}} = \left\langle \cos^2(\phi)N_{F=1}^+ + \frac{4}{3}\sin^2(\phi)N_{F=2}^+ \right\rangle . \quad (5.3)$$

Here, equation (5.3) is evaluated for a coherent state and $\hat{N}_{F=1,2}^+$ denotes the population of the sidemodes in each magnetic hyperfine state

$$\begin{aligned} N_{F=1}^+ &= N_{-1}^{F=1} + N_{+1}^{F=1} \\ N_{F=2}^+ &= N_{-2}^{F=2} + N_{+2}^{F=2} . \end{aligned} \quad (5.4)$$

The normalization can be calculated using (2.8), which is the binomial distribution of the coherent polar state. A short calculation is given in appendix C.1. With this

choice our analysis signals squeezing for

$$\frac{\Delta^2 F(\phi)}{\langle N(\phi) \rangle_{\text{CS}}} < 1 . \quad (5.5)$$

Here, it useful to go back to figure 3.3, where we showed the results of the imaging calibration. There, we prepared a coherent state and measured its mean atom number and variance. Evaluating the fluctuations as a function of the atom number gave us a linear dependency with a slope of 1. In the calibration we took special care to get a slope of 1. Therefore we know, that the ratio in equation (5.5) is equal to 1 for a coherent state in our experiment.

When evaluating the variance as a function of the angle ϕ we expect a sinusoidal fringe for a squeezed state, with its minimum below 1. We denote the corresponding angle as the squeezing angle $\phi_{\min} = \phi_{\text{sq}}$. Additionally, we detect enhanced fluctuations with a $\pi/2$ shift, i.e. $\phi_{\max} = \phi_{\min} + \pi/2$. To determine the uncertainty of our evaluation we use the jackknife method [36]. A brief summary is found in section C.2.

To elucidate this analysis we pick up again the data generated with truncated Wigner simulations presented in section 2.2.3 and summarized in figure 2.4. Now, we analyze them using the variance analysis described above. For that we evaluate the normalized fluctuations $\Delta^2 F(\phi) / \langle N(\phi) \rangle$ as a function of the angle ϕ . Then, we obtain the fringes displayed in figure 5.1, where $\pi = 0$ and $\pi = 1/2$ represent S_x and Q_{yz} , respectively. The shaded area represents one standard deviation of the normalized fluctuations. We see that the polar state (A) does not have a pronounced structure and features approximate constant fluctuations in any direction. Specifically, this state shows no reduced fluctuations. Thus, any residual fluctuations are classical and appear due to sampling effects. In contrast to the polar state, the state after 100 ms time evolution shows a rich structure in the variance analysis (see figure 5.1 B). On the one side, we see a pronounced maxima at $\approx 0.4\pi$ corresponding to the long axis and on the other side, we extract the minimum at $\approx 0.9\pi$. At this minimum the normalized fluctuations are below one and thus signals squeezing. Therefore, this state indeed features squeezing, as our first qualitative impression in figure 2.4 B indicated. Further, this analysis allows us to estimate the number of participating atoms in the spin-mixing process by evaluating the maximal variance. After an evolution time of 100 ms we find this value to be ≈ 16 . Evaluating the variance for the longest evolution time, yields the fringe in figure 5.1 C. We still see a sinusoidal behavior, that however features enhanced fluctuations (> 1) in all directions. As discussed before, the truncated Wigner simulation does not capture this state anymore and using our readout scheme, we do not expect to neither detect squeezing nor entanglement after these long evolution times.

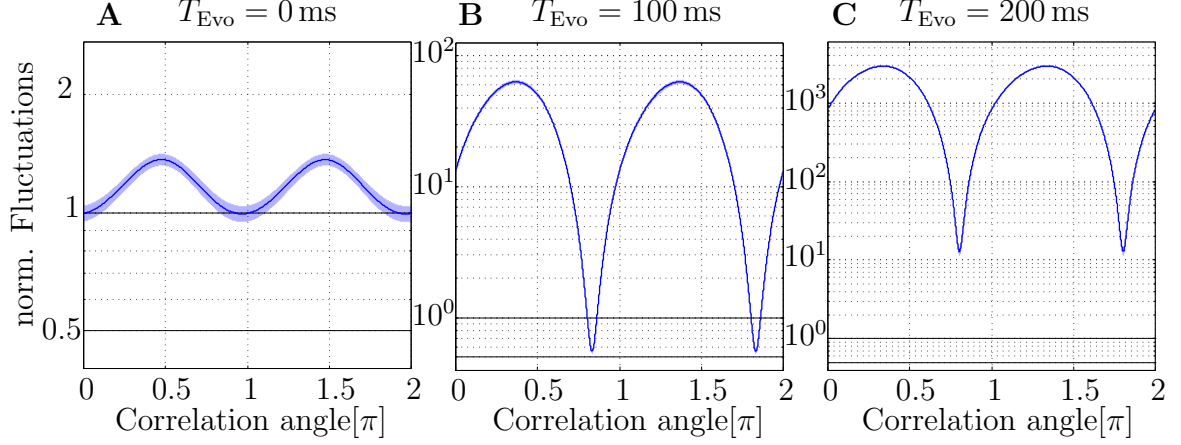


Figure 5.1: Variance analysis: Variance analysis for the data from figure 2.4. (A) The initial state, which is a coherent state, shows nearly no correlation angle dependence in the fluctuation and has normalized fluctuations of ≈ 1 . Any residual structure is due to numerical inaccuracies. (B) After 100 ms we get a pronounced minimum (≈ 0.5) at $\approx 0.75\pi$, which is the fundamental limit due to the measurement setup. (C) As the state follow the classical trajectories it becomes non Gaussian and therefore the fluctuations increase and the squeezing vanishes.

5.2 Duan criterion as a entanglement witness

With the newly introduced observable $F(\phi)$ we are able to detect squeezing in any direction of the phase space spanned by S_x and Q_{yz} , cf. figure 5.1. Now we use the corresponding operator to introduce an entanglement witness, which allows to detect entanglement between the left and the right half of the atom cloud. This witness is based on the Duan criterion [37] and uses the Arthur-Kelly [38] inseparability criterion for joint measurements. Here, we only give a brief motivation. A detailed calculation can be found in [14, 39]. We start with the general case, where we have two Hermitian operators $\hat{F}^k(\phi)$ acting on subsystem $k \in \{L, R\}$ and define

$$\begin{aligned}\hat{F}_1 &= \hat{F}^L(\phi_{\min}) + \hat{F}^R(\phi_{\min}) \\ \hat{F}_2 &= \hat{F}^L(\phi_{\min} + \pi/2) - \hat{F}^R(\phi_{\min} + \pi/2) .\end{aligned}\tag{5.6}$$

Here, the “-” in the second line accounts for the structure of the first excited mode of the effective potential, cf. figure 3.4. With Heisenberg’s uncertainty relation and the Cauchy-Schwarz inequality we find for any separable state the following inequality

$$\begin{aligned}\Delta^2 \hat{F}_1 + \Delta^2 \hat{F}_2 &\geq \left| \left\langle \left[\hat{F}^L(\phi_{\min}), \hat{F}^L(\phi_{\min} + \pi/2) \right] \right\rangle \right| \\ &\quad + \left| \left\langle \left[\hat{F}^R(\phi_{\min} + \pi/2), \hat{F}^R(\phi_{\min} + \pi/2) \right] \right\rangle \right| .\end{aligned}\tag{5.7}$$

Here, we have to consider, that we apply a simultaneous readout of two non-commuting observables. According to Heisenberg it is not possible to measure two

non-commuting observables with unlimited precision. Thus, this method introduces some additional noise, that leads to a modified uncertainty relation. We adapt our observables by introducing an uncertainty δF_i , $i \in \{1, 2\}$ caused by the simultaneous readout, which yields $\tilde{F}_i = F_i + \delta F_i$. As the readout measures two orthogonal observables their commutator vanishes and thus we get [39]

$$0 = \langle [\tilde{F}_1, \tilde{F}_2] \rangle \Rightarrow |\langle [F_1, F_2] \rangle| = |\langle [\delta F_1, \delta F_2] \rangle|. \quad (5.8)$$

We then arrive at the uncertainty relation for a joint measurement

$$\Delta^2 \tilde{F}_1 \cdot \Delta^2 \tilde{F}_2 \geq |\langle [F_1, F_2] \rangle|^2, \quad (5.9)$$

which introduces a factor of two on the right hand side due to the additional uncertainty of the readout method, compared with the well known Heisenberg uncertainty relation. Applying this uncertainty relation to the derivation of the the Duan criterion from equation (5.7), adapts the inequality to our measurement method and yields

$$\begin{aligned} \Delta^2 \hat{F}_1 + \Delta^2 \hat{F}_2 \geq 2 \left| \left\langle \left[\hat{F}^L(\phi_{\min}), \hat{F}^L(\phi_{\min} + \pi/2) \right] \right\rangle \right| \\ + \left| \left\langle \left[\hat{F}^R(\phi_{\min} + \pi/2), \hat{F}^R(\phi_{\min} + \pi/2) \right] \right\rangle \right|. \end{aligned} \quad (5.10)$$

Using the definition of F , cf. equation 5.1 and the commutation relation $[S_x, Q_{yz}] \approx 2iN$, cf. equation (2.37) gives

$$[F^k(\theta_{\min}), F^k(\theta_{\min} + \pi/2)] \approx 2iN_k, \quad (5.11)$$

with $k \in \{L, R\}$. For an equal atom number in both halves we find $N_k = N/2$. Inserting into equation (5.10) yields our adapted Duan criterion.

$$D = \Delta^2 (F^L(\theta_{\min}) + F^R(\theta_{\min})) + \Delta^2 (F^L(\theta_{\min} + \pi/2) - F^R(\theta_{\min} + \pi/2)) \geq 2N. \quad (5.12)$$

Before we apply the analysis method, we investigate some noise sources and their behavior on the readout scheme with truncated Wigner simulations.

5.3 Truncated Wigner simulations of spin-nematic squeezing

So far, we looked at three specific times and investigated the phase space behavior, cf. figure 2.4, and the variance analysis, cf. figure 5.1, using truncated Wigner simulations. This gave us a good impression of the ongoing dynamics and a first insight, that long times are unsuitable for the detection of squeezing. In the following paragraphs we look into potential noise sources interfering with our readout scheme.

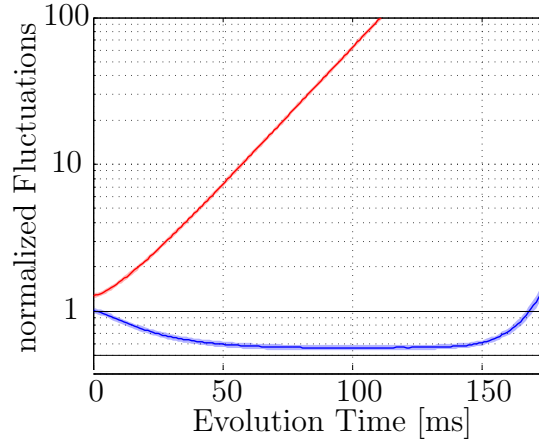


Figure 5.2: Time Evolution: For every time step we take the minimal (blue) and maximal (red) fluctuations. This gives us an estimation of the fluctuations and if we expect squeezing at some time. This leads to a faster analysis for later simulations. For the ideal situation, i.e. no noise effects, we see that the minimum is below 1 up to an evolution time of ≈ 150 ms. Here, for evolution times between 50 ms and 150 ms the minimal fluctuations are constant at the fundamental limit.

5.3.1 Evaluation of truncated Wigner simulation data

To get a better overview over the time dynamics we choose a different form of representation. We extract for each time step the minimum and maximum variance and plot it as a function of time. This allows to estimate the fluctuation and in particular to check if we expect squeezing at some time. Such a time evolution is shown in figure 5.2. The minimal variance is represented by the blue line. We see, that the initial state has normalized fluctuations of ≈ 1 , which we have seen in figure 5.1 A. As the state evolves, the minimum decreases till 50 ms, where it reaches the fundamental limit of 0.5. After 150 ms the minimal fluctuations increase, which signals the transformation to a non-Gaussian state. We also see the rapidly increasing maximum, which is due to more participating atoms at longer times. Thus, this evolution conveniently shows, that we expect to measure squeezing between 50 and 100 ms. Therefore, we use this representation throughout the further investigations of external noise effects.

5.3.2 Probing limitations of readout scheme with truncated Wigner simulations

Having found a convenient representation, we now investigate how experimental inaccuracies such as phase fluctuations or changes in the detuning influence the readout. In figure 5.2, where we show the time evolution of the fluctuations for an ideal system, we see a high squeezing up to ≈ 150 ms. As we expect that noise effects will decrease the capability of detecting squeezing we restrict our time evolution to 150 ms in the following discussion. We start our discussion with effects of Larmor

and Spinor phase fluctuations.

Larmor phase fluctuations Fluctuations in the rotation axis of the mw split pulses can occur, due to phase fluctuations in the mw setup. These translate into a Larmor or Spinor phase fluctuation and thus the observables that are measured change. To study the effect of a fluctuating Larmor phase, we pick a random Larmor phase $\delta\phi_L$ from a normal distribution with a fixed standard deviation in each realization. This leads to a change in the rotation axis, which can be expressed using the mw transition operators

$$\hat{C}_y^{m_1, m_2} \rightarrow \cos(\phi_L + \delta\phi_L) \hat{C}_y^{m_1, m_2} + \sin(\phi_L + \delta\phi_L) \hat{C}_x^{m_1, m_2}, \quad (5.13)$$

Here, $\hat{C}_i^{m_1, m_2}$, with $i \in \{x, y\}$, denotes the operator for a mw transition between the magnetic substates m_1 and m_2 introduced in section 3.3.2. The subindex i refers to the rotation axis of the pulse. Then, the sequence splitting the population can be written as

$$\begin{aligned} |\psi_{\text{split}}\rangle &= e^{\cos(\delta\phi_L) \hat{C}_y^{-1, -1} + \sin(\delta\phi_L) \hat{C}_x^{-1, -1}} \\ &\quad \cdot e^{\hat{C}_x^{0, 0}} \\ &\quad \cdot e^{\cos(-\delta\phi_L) \hat{C}_y^{+1, +1} + \sin(-\delta\phi_L) \hat{C}_x^{+1, +1}} |\psi_{\text{TW}}\rangle. \end{aligned} \quad (5.14)$$

Here, $|\psi_{\text{TW}}\rangle$ denotes the state directly after the truncated Wigner time evolution. Further, we set $\phi_L = 0$, which is guaranteed due to our phase calibration measurement. The second line represents the split pulse, that is employed on the state $|1, 0\rangle$. The first and third row show the pulses on the $m_F = \pm 1$ states, where the rotation axis depends on the Larmor phase, now. Due to the definition of the Larmor phase the sign of $\delta\phi_L$ changes.

We apply truncated Wigner simulations with Larmor phase noises sampled from a normal distribution with different standard deviations. The results are shown in figure 5.3. Subplot B and C show the time evolution with a fixed standard deviation. We see, that in both cases the fundamental limit of 0.5 is not reached. Besides increased fluctuations (still below 1) the behavior changes with increasing noise as the time span, where the minimum is reached, decreases. To provide a different point of view, we show the evolution of the minimal variance as a function of the noise, see figure 5.3 A. Here, we fix an evolution time of $T_{\text{Evo}} = 100 \text{ ms}$ and increase the Larmor phase noise. While for small fluctuations we still can measure the fundamental limit, our ability to detect squeezing decreases, with increasing fluctuations. Above a standard deviation of 0.2π we reach the classical limit and thus are not able to detect squeezing anymore.

Spinor and Larmor phase fluctuations Besides the Larmor phase, we also imprint a Spinor phase during the split sequence with the mw setup. Thus, we now analyze the effects when considering phase fluctuations in all mw pulses. The additional

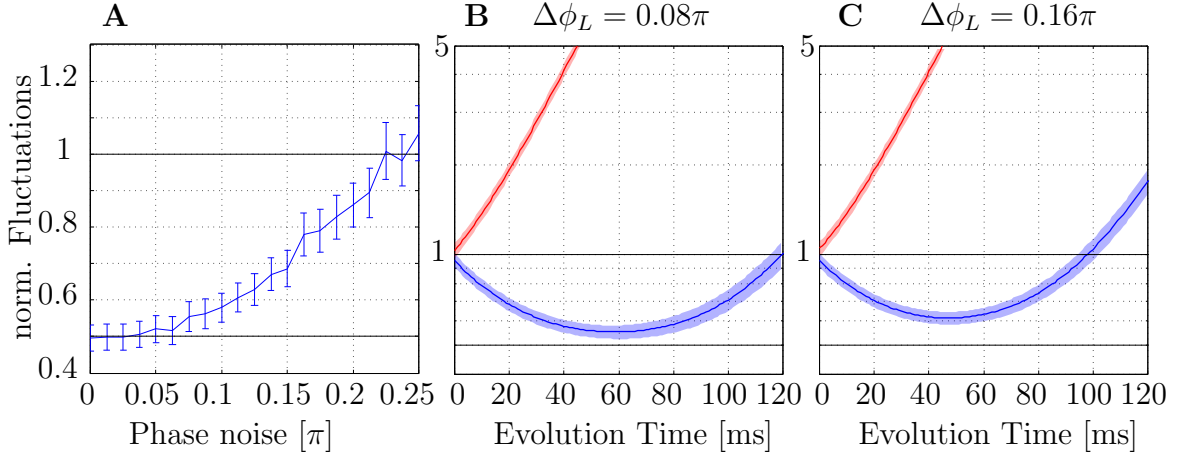


Figure 5.3: Larmor phase noise: We employ a truncated Wigner simulation which randomly changes the Larmor phase and therefore the rotation axis of the two mw split pulses from $m_F = \pm 1$. **(A)** For different standard deviations of the phase noise we detect the minimum variance after an evolution time of $T_{\text{evo}} = 100$ ms. While high phase fluctuations lead to a variance on the order of that of a coherent state, small phase noises still allow to detect squeezing. **(B)** We set the standard deviation to $\approx 0.04\pi$. Here, the minimal fluctuations clearly increase earlier compared with the results for no Larmor phase noise. **(C)** Increasing the noise further to $\approx 0.08\pi$ results in a higher minimum ≈ 0.6 at $\tau_{\text{Evo}} = 45$ ms and a shorter time where squeezing is expected.

Spinor phase noise $\delta\phi_S$ is accounted for by changing the operator in the second line of equation (5.14)

$$\begin{aligned} \hat{C}_x^{0,0} &\rightarrow \cos(\phi_S + \delta\phi_S)\hat{C}_x^{0,0} + \sin(\phi_S + \delta\phi_S)\hat{C}_y^{0,0} \\ &\stackrel{\phi_S=0}{=} \cos(\delta\phi_S)\hat{C}_x^{0,0} + \sin(\delta\phi_S)\hat{C}_y^{0,0}. \end{aligned} \quad (5.15)$$

Here, we set $\phi_S = 0$ because of the calibration measurement as explained before and $\delta\phi_S$ is drawn from a normal distribution with a fixed standard deviation. Compared with equation (5.13) the indices x and y are exchanged due to the $\pi/2$ Spinor phase shift from the readout sequence. With this, we can employ a simulation and we get the results shown in figure 5.4, where we use the same representation and values as in the discussion of the Larmor phase fluctuations. We see again, that with increasing fluctuations our ability to detect squeezing decreases. As now all three pulses are fluctuating, the classical limit is reached earlier at a standard deviation of $\approx 0.15\pi$, cf. figure 5.4 A. This is also indicated in figure 5.4 B and C, where we see the time evolution, that shows a smaller time span of squeezing for high fluctuations compared with only the Larmor noise.

In the previous two paragraphs we have seen that high fluctuations in the mw setup lead to an averaging of classical correlations, that decrease the ability to detect squeezing. However, we investigated the behavior up to rather high fluctuations. As demonstrated before, cf. figure 3.6, we have a high phase stability from our mw setup

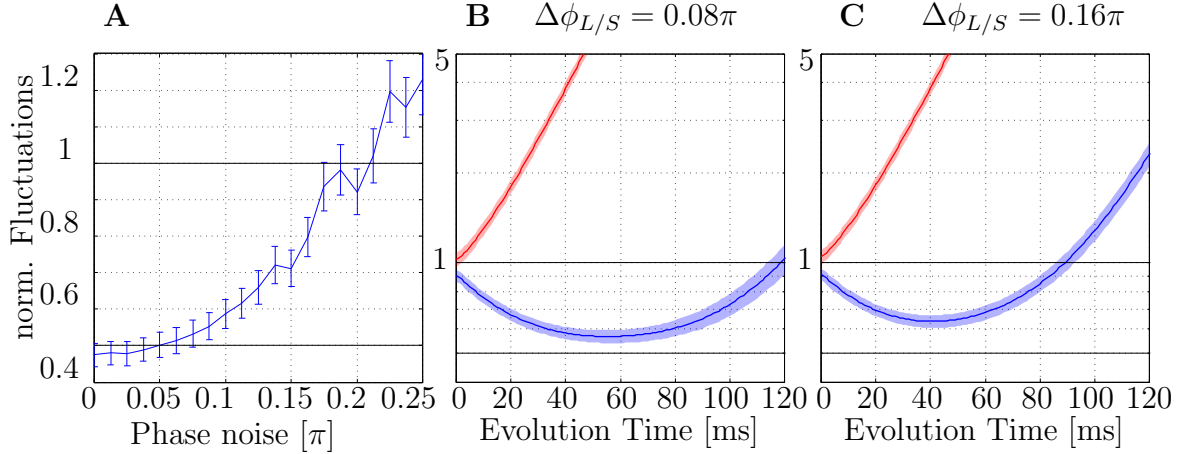


Figure 5.4: Spinor and Larmor phase noise: We considered phase fluctuations in all three mw-split pulses, i.e. Spinor and Larmor phase fluctuations.

and thus expect no significant reduction by the mw setup.

q-fluctuations During our experiments, we regularly make a spectroscopy measurement, cf. section 3.3.4. From these measurements, we observe that our spectroscopy shifts over time, i.e. the resonant q_{eff} value changes. Thus, for a measurement we do not stay on resonance for the spin changing collisions and therefore we investigate the effects of a q shift on our detection. To get a feeling for the behavior, in case q changes, we employ a truncated Wigner simulation with $q \in \{1.5, 2.5\}$ Hz instead of $q = 2$ Hz as before. The corresponding time evolution is shown in figure 5.5 A and B. We see that for higher values of q we get higher fluctuations compared with lower values of q and the original simulation. However, both simulations still show a stable regime with low fluctuations, where we expect to detect squeezing.

The effect of a shift on the trajectories can also be observed in figure 2.3. There, the separatrix has a different angle depending on q . To study the effect of a q shift, we employ a simulation, that implements a linear shift in q over $\delta q = 0.5$ Hz. As a starting value we chose $q = 2$ Hz and linearly decrease it to $q = 1.5$ Hz. The simulation results are shown in figure 5.5 C. Here, we see that besides overall higher fluctuations we get a shorter time, where these small fluctuations are reached. To overcome this behavior in the measurement we still could limit our data set on a specific time in the post analysis with the downside of losing some statistics.

Throughout the previous paragraphs, we have seen that we need a high degree of control as any fluctuations translate into less detectable squeezing. However, as we discussed in sections 3 and 4 our experimental setup is optimized and has low noise inputs. Thus, the fluctuations in the simulations are expected to be overestimated and for moderate fluctuations the simulations still signal squeezing. With these insights into the readout scheme and its capabilities as well as our analysis method, we now turn to the discussion of our experimental results.

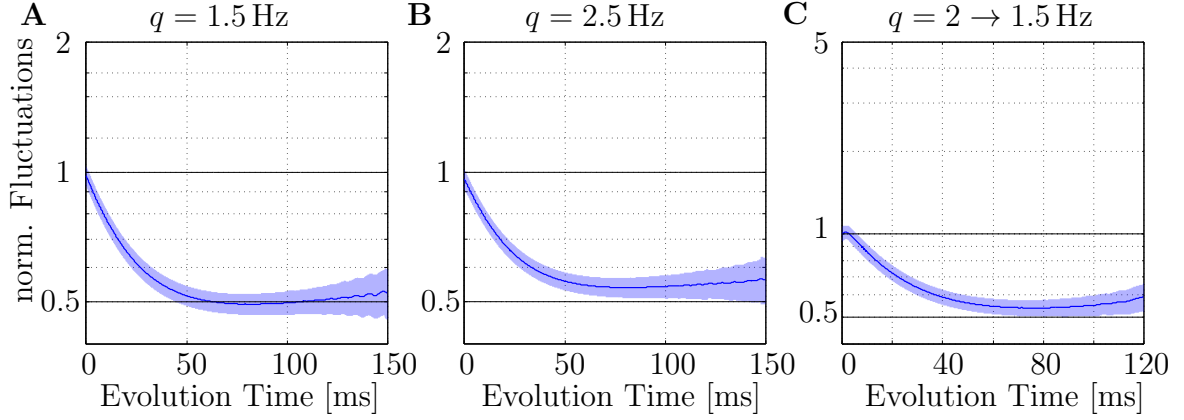


Figure 5.5: Change of detuning: We simulated the evolution for different values of $q \in \{1.5, 2.5\}$ Hz. (A) Minimal fluctuations as a function of time for a lower detuning than in the experimental setting $q = 1.5$ Hz. (B) Time evolution for $q = 2.5$ Hz, which is a higher value than in the experiment. (C) After an observation of an approximate linear shift in the detuning, we implement this shift from $q = 2$ Hz to $q = 1.5$ Hz.

5.4 Experimental results

5.4.1 Spin-nematic squeezing and entanglement

Having all the methods at hand we employed spin-changing collisions for 100 ms in our experiment and measured the atomic cloud, from which we extract the observables S_x and Q_{yz} . By repeating the experiment multiple times, we are able to sample their distribution of the phase space spanned by these two observables. We evaluate these observables, by integrating over the spatial extension of the cloud. As we have access to the spatial degree of freedom, we are able to split the cloud into two halves, left (L) and right (R) in the post analysis. Evaluating different combinations, i.e. $L \pm R$, yield the phase spaces shown in figure 5.6 A (upper row). Here, we normalized the observables to $\sqrt{N_F^+}$. Both combinations lead to an elliptical phase space distribution, that is a first indication of squeezing. However, we can also evaluate both halves individually, which corresponds to tracing out the other subsystem. This gives the phase spaces shown in figure 5.6 A (lower row), where we see an isotropic distribution. Further, we can quantify this, by evaluating $\Delta^2 F^{L/R}(\phi)$, which results in the plots shown in figure 5.6 B. These validate the first impression, that both subsystems have enhanced but isotropic fluctuations. Both fringes are above the coherent state fluctuations of 1 and do not show a pronounced dependence on the correlation angle ϕ as they vary not much around 2.

However, to certify entanglement, we have to look at the composition between the two halves and not both halves individually. Thus, we evaluate the fluctuations $\Delta^2 F^{L \pm R}(\phi)$ for $L \pm R$. This yields the fringes in figure 5.7, where $L \pm R$ is shown in blue and red, respectively. We see, that both analyses show squeezing as their

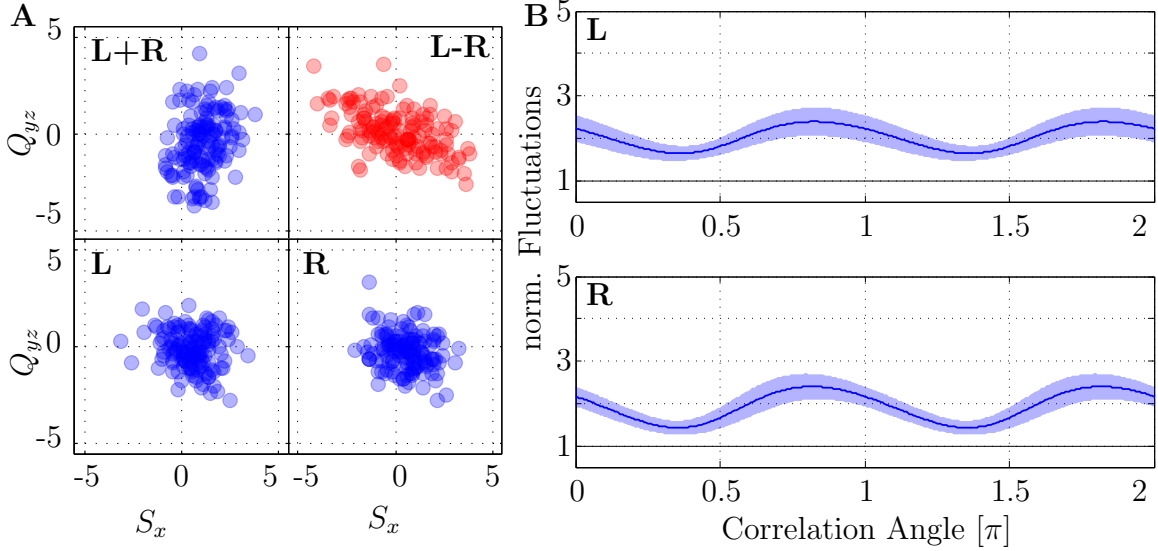


Figure 5.6: Phase-space and fluctuations (A) Phase-space for different spatial decompositions as depicted in the upper corners. While in the upper row, the phase space form a squeezing ellipse, in the lower row we see a coherent state-like distribution with enhanced fluctuations in the single halves. (B) Evaluation of normalized fluctuations for the left and right half separately. The quantitative analysis shows enhanced fluctuations for both halves.

minimum is below 1. From these minima, we can infer the squeezing value [39]

$$\xi = 10 \cdot \log_{10} [2 (\Delta^2 F(\phi_{\min}) - 0.5)] , \quad (5.16)$$

where for a coherent state $\xi_{\text{CS}} = 0$ dB. The measurement data from figure 5.7 yield an inferred squeezing value of $\xi_{\text{L+R}} = -3.4^{+1.4}_{-2.0}$ dB and $\xi_{\text{L-R}} = -2.8^{+1.6}_{-2.7}$ dB, respectively, which is significantly below zero.

To certify entanglement, we use the inequality given in equation 5.12. Thus, we extract

$$\begin{aligned} \Delta^2 (F^{\text{L}}(\theta_{\min}) + F^{\text{R}}(\theta_{\min})) &= 0.73 \pm 0.09 \\ \Delta^2 (F^{\text{L}}(\theta_{\min} + \pi/2) - F^{\text{R}}(\theta_{\min} + \pi/2)) &= 0.77 \pm 0.12 \end{aligned} \quad (5.17)$$

Inserting both results into equation (5.12) yields for the left hand side 1.50 ± 0.14 , which is below 2. Therefore, the violation of the imbalance is given by -0.5 ± 0.14 , and thus detects entanglement between the left and the right half of the atom cloud.

5.4.2 Squeezing in a box-like potential

The system, which yielded the above results, has been employed in a tightly confined harmonic trap. In this trap, the modes were equally spaced and we could

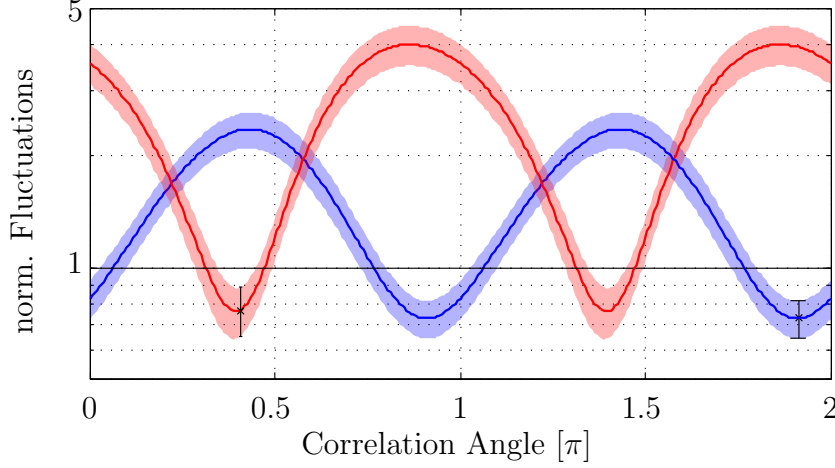


Figure 5.7: Squeezing and entanglement detection Evaluation of $F^L(\phi) + F^R(\phi)$ (blue) and $F^L(\phi) - F^R(\phi)$ (red). The shaded area represents the one standard deviation statistical error. The two points (black) indicate $\phi_{\min}$ and $\phi_{\min} + 3\pi/2$, respectively. Both evaluations show squeezing, cf. figure 5.6 A.

bring two of these modes simultaneously into resonance. In section 3.4 we mentioned our capabilities to create a box-like potential using our arbitrary waveform generator in combination with an AOD. Due to broader extension of the box-like potential the energy difference of the modes is much smaller. Thus, by employing microwave dressing we could tune several modes simultaneously into resonance with the spin-mixing dynamics. In this setup we detect simultaneous squeezing in the four energetically lowest modes, see [39]. We present the evaluation results of these 4 modes in figure 5.8. For the evaluation we use the wavefunctions of a box potential

$$f(x) = \sin\left(\frac{2\pi n}{L}x\right). \quad (5.18)$$

Here, n denotes the number of the mode and L the size of the evaluation region. The evaluation of the observables $O \in \{S_x, Q_{yz}\}$ with these functions is done by calculating $\mathcal{O} = \sum_x O(x) \cdot f(x)$. This yields for each mode the normalized fluctuations presented in figure 5.8. In each mode we find a minimum value below one and thus a signal of squeezing. We infer the following squeezing values for the different modes n

$$\begin{aligned} n = 1, & \quad \xi = -3.8_{-2.8}^{+1.7} \text{ dB} \\ n = 2, & \quad \xi = -6.0_{-5.0}^{+2.3} \text{ dB} \\ n = 3, & \quad \xi = -4.5_{-3.6}^{+2.0} \text{ dB} \\ n = 4, & \quad \xi = -8_{-14}^{+3} \text{ dB} \end{aligned}$$

Having detected squeezing in a box-like potential, this system is also suitable to study multimode dynamics.

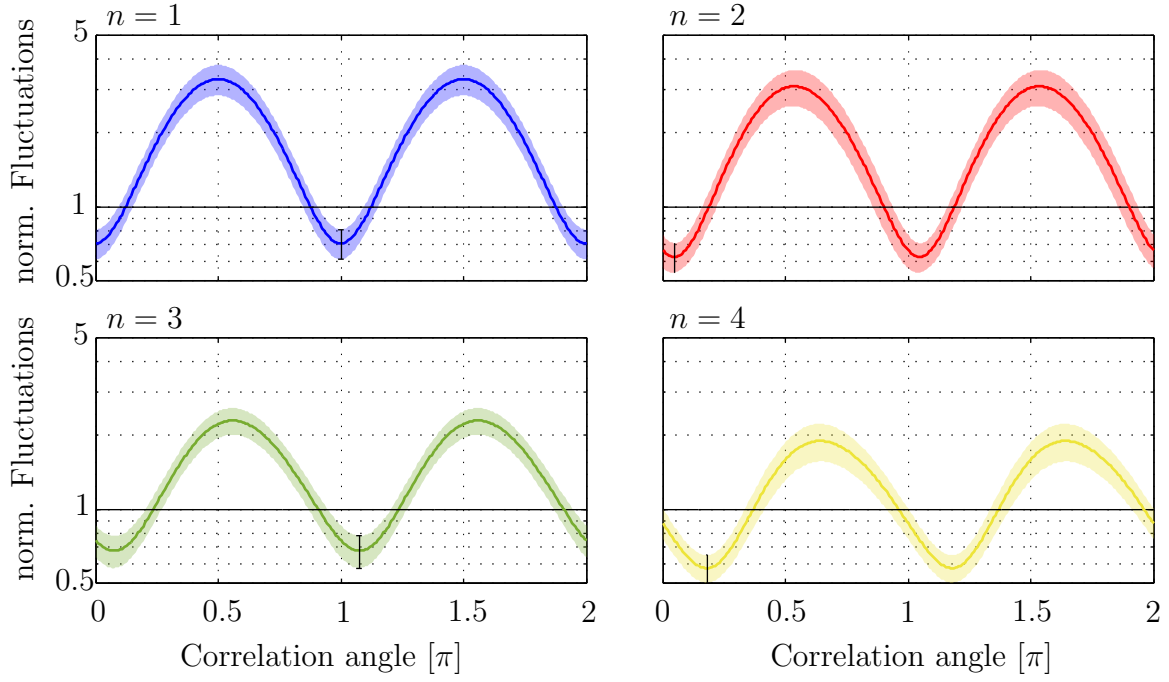


Figure 5.8: Squeezing in a box-like potential We initiate spin-mixing dynamics by tuning the 4 energetically lowest modes into resonance. We evaluate the modes using the wavefunctions of a box potential. The corresponding mode is indicated in the figure. For each evaluation we find squeezing (black marker).

6 Outlook

In the previous chapters we illuminated our scheme to employ spin-changing collisions and to simultaneously measure two orthogonal observables allowing to detect entanglement. We started by developing a basic understanding of the theoretical concepts of our experimental spin-1 platform. Here, we focused on the graphical representation using the spin and spin-nematic sphere. Then, we investigated how internal spin-spin interactions give rise to squeezing. To further describe this phenomenon we chose the harmonic oscillator. This allowed us to investigate squeezing by looking at the uncertainty relation of conjugate observables. We found, that fluctuations below the ones of a coherent state signal squeezing. A coherent state is a special state, as it has a symmetric probability distribution in two orthogonal observables and saturates Heisenberg's uncertainty relation. This gave us the possibility to describe spin squeezing by putting the spin fluctuations in relation to those of a coherent state.

The detection of squeezing is a sensitive task. For example, in our experiments only around 8 atoms out of 4×10^4 contribute to the squeezing signal. Thus, our experimental setup has to be very precise and should not introduce any external noise effects. Besides having already high control we further improved the setup with new arbitrary waveform generators. We explained our methods to manipulate the system and to image the atoms with high precision. To reach this level we summarized our methods to calibrate the setup. Having this control over the experiment allows us to realize a joint measurement of the two spin-1 observables S_x and Q_{yz} . The idea behind this scheme is to use an additional hyperfine manifold, which serves as an auxiliary system.

To study the dynamics and the capabilities of the readout, we used truncated Wigner simulations. These simulations sample starting points from the distribution of a coherent state and evolve it by using the classical equation of motions. Especially in our regime of high atom numbers and short evolution times, these simulations capture the dynamics of the experiment well. To detect squeezing we introduced our method, which is a homodyne-like detection scheme. This allows to observe the small signal coming from the spin-mixing dynamics. To finally detect entanglement between two spatially distinct regions of our cloud, i.e. left and right half, we adapted the Duan criterion to joint measurements. We then used our squeezing measurement to certify entanglement between the two halves.

Having a high control over the experiment and the readout allows to further investigate the system. In the following a few examples are given for which the setup in this thesis may serve as a foundation.

Entanglement structure During the discussion of the results we used the adapted Duan criterion. We specifically evaluated the criterion for the case, where our two analysis methods, i.e. $F^L(\phi) \pm F^R(\phi)$, are separated by $\Delta\phi = \pi/2$. However, our readout gives us the possibility to evaluate every angle ϕ individually in post analysis. Thus we can analyze $\Delta^2 F^{L/R}(\phi_{L/R})$ for every angle $\phi_{L/R}$. This gives us a

map-like analysis, that shows the fluctuations of $F^{L/R}(\phi_{L/R})$. From this evaluation we then take any two points and can determine the corresponding fluctuations. To certify entanglement however, we still have to consider, that the commutator of the observable $[F^L(\phi), F^R(\phi + \delta\phi)] \approx 2iN \sin(\delta\phi)$ depends on $\delta\phi$. The commutator is used in the calculation of the Duan criterion, cf. equation (5.10), and thus changes the inequality. Considering the more general commutator we can extract a map-like entanglement matrix. For better visibility it is useful to fix $\delta\phi_{L/R}$, which is the angle difference between the two points and thus yield a constant commutator. To extract the relevant violation of the witness we take the maximum over the violation of the witness. With this we can extract an entanglement structure. Here, we make use of our spatially resolved joint measurement, that allows to process all these steps in post analysis. A more rigorous treatment is given in [39]. This evaluation enables to detect a new entanglement space and to study the buildup of entanglement in a new perspective.

Extracting the entropy When discussing entanglement, especially in terms of quantum information, entropies play an important role. One of those, is the so-called Wehrl entropy which is defined as [40]

$$S_W(\rho) = - \int Q_\rho(S_x, Q_{yz}) \log Q_\rho(S_x, Q_{yz}) dS_x dQ_{yz} . \quad (6.1)$$

Here, $Q_\rho(S_x, Q_{yz})$ is the Husimi function. For the phase space spanned by S_x and Q_{yz} the Husimi function reads [14, 41]

$$Q_\rho(S_x, Q_{yz}) \propto \sum_{\alpha} \langle S_x, Q_{yz} | \hat{\rho} | S_x, Q_{yz} \rangle_{\alpha} . \quad (6.2)$$

In our case, i.e. the undepleted pump region our measurement realizes a direct sampling from the Husimi distribution and thus would allow to evaluate the Wehrl entropy. However, effects like finite measurement statistics and the binning of the measurement data, still remain unexplored.

Quantum computing The past years have brought many experimental breakthroughs in the area of quantum computing using trapped ions [42] and superconducting circuits [43]. Coming from a classical computer with a two level bit, the main aspect for these type of quantum computers is to replace the bit with a quantum mechanical two-level system, i.e. a qubit [44]. These types of quantum computers build on unitary manipulations (so-called quantum gates). The driving mechanism in quantum computers is entanglement [45]. This allows them to solve problems beyond classical computers [46].

Having a Bose-Einstein condensate realizes a continuous variable system and thus the approach to build a quantum computer using quantum gates, is not suitable. Therefore, new proposals have been made. One of them being the one-way quantum

computer [47]. In contrast to a universal quantum computer, which works gate by gate, a one-way quantum computer is dedicated to a single problem only. Here, the “algorithm” is imprinted onto a so-called cluster state, which is represented by different subsystems. These, cluster states are highly entangled states. A successive measurement of the subsystems and unitary operations depending on the measurement outcome of the previous subsystem, then realize a quantum computation.

A Spin-1 and variance

In the following paragraphs we give some additional information about the spin system description. We present some additional operators for completeness, that we neglected in the main text.

A.1 Spin-1 operators

In the main text we have given the relevant spin-1 operators. Here, we give the spin and quadrupole operators that form a basis for $SU(3)$. Additionally, we give their representation in second quantization.

Matrix notation:

$$\hat{S}_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\hat{S}_y = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$\hat{S}_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

$$\hat{Q}_{xz} = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & -1 \\ 0 & -1 & 0 \end{pmatrix}$$

$$\hat{Q}_{yz} = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & i \\ 0 & -i & 0 \end{pmatrix}$$

$$\hat{Q}_{zz} = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{4}{3} & 0 \\ 0 & 0 & \frac{2}{3} \end{pmatrix}$$

$$\frac{1}{2} (\hat{Q}_{xx} - \hat{Q}_{yy}) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

$$\hat{Q}_{xy} = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

Second-quantization:

$$\hat{S}_x = \frac{1}{\sqrt{2}} \hat{a}_0^\dagger (\hat{a}_{+1} + \hat{a}_{-1}) + \frac{1}{\sqrt{2}} (\hat{a}_{+1}^\dagger + \hat{a}_{-1}^\dagger) \hat{a}_0$$

$$\hat{S}_y = \frac{i}{\sqrt{2}} \hat{a}_0^\dagger (\hat{a}_{+1} - \hat{a}_{-1}) - \frac{i}{\sqrt{2}} (\hat{a}_{+1}^\dagger - \hat{a}_{-1}^\dagger) \hat{a}_0$$

$$\hat{S}_z = \hat{a}_{+1}^\dagger \hat{a}_{+1} - \hat{a}_{-1}^\dagger \hat{a}_{-1}$$

$$\hat{Q}_{xz} = \frac{1}{\sqrt{2}} \hat{a}_0^\dagger (\hat{a}_{+1} - \hat{a}_{-1}) + \frac{1}{\sqrt{2}} (\hat{a}_{+1}^\dagger + \hat{a}_{-1}^\dagger) \hat{a}_0$$

$$\hat{Q}_{yz} = \frac{i}{\sqrt{2}} \hat{a}_0^\dagger (\hat{a}_{+1} + \hat{a}_{-1}) - \frac{i}{\sqrt{2}} (\hat{a}_{+1}^\dagger + \hat{a}_{-1}^\dagger) \hat{a}_0$$

$$\hat{Q}_{zz} = \frac{2}{3} (\hat{a}_{+1}^\dagger \hat{a}_{+1} + \hat{a}_{-1}^\dagger \hat{a}_{-1}) - \frac{4}{3} \hat{a}_0^\dagger \hat{a}_0$$

$$\frac{1}{2} (\hat{Q}_{xx} - \hat{Q}_{yy}) = \hat{a}_{+1}^\dagger \hat{a}_{-1} + \hat{a}_{-1}^\dagger \hat{a}_{+1}$$

$$\hat{Q}_{xy} = i \hat{a}_{+1}^\dagger \hat{a}_{-1} - i \hat{a}_{-1}^\dagger \hat{a}_{+1}$$

A.2 Spin-2 operators

For completeness, we give the relevant spin-2 operators here

$$\begin{aligned}
 \hat{S}_x^{F=2} &= \begin{pmatrix} 0 & 2 & 0 & 0 & 0 \\ 2 & 0 & \sqrt{6} & 0 & 0 \\ 0 & \sqrt{6} & 0 & \sqrt{6} & 0 \\ 0 & 0 & \sqrt{6} & 0 & 2 \\ 0 & 0 & 0 & 2 & 0 \end{pmatrix}, & \hat{S}_y^{F=2} &= \begin{pmatrix} 0 & 2i & 0 & 0 & 0 \\ 2i & 0 & \sqrt{6}i & 0 & 0 \\ 0 & \sqrt{6}i & 0 & \sqrt{6}i & 0 \\ 0 & 0 & \sqrt{6}i & 0 & 2i \\ 0 & 0 & 0 & 2i & 0 \end{pmatrix}, \\
 \hat{S}_z^{F=2} &= \begin{pmatrix} 2 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 2 \end{pmatrix}.
 \end{aligned} \tag{A.1}$$

These fulfill the SU(2) commutation relation $[\hat{S}_i, \hat{S}_j] = \epsilon_{ijk} \hat{S}_k$ and describe (rf) spin rotations in $F = 2$.

B Programming an arbitrary waveform generator

In the main text we mentioned the two setups, where we used an arbitrary waveform generator (AWG). In the following chapter we discuss the programming of the AWG. The setup in our experiment is used to discuss the methods and to give some numbers, but the general ideas behind it can be applied to any type of arbitrary waveform generator. Having the great capability of defining a voltage value at each sample comes with the challenge of a limited memory of 64 MSa. A helpful function of the Keysight devices is the replay mode. Here, a pulse segment is repeated a fixed number of times before playing the subsequent segment. Thus, this mode is useful especially for periodic signals. Before turning to the calculation of the repetitions needed to create the wanted signal, we have to consider some technical issues. In the following we show the key aspects of our code and highlight the calculations to obtain a good balance between memory consumption and a high sample rate.

To create a sequence for the AWG a user of the device outlines it by giving a few parameters. These are:

- The type of the pulse, e.g. “sinus”, “square”, “constant”
- The duration τ of the pulse
- The frequency f
- The peak-to-peak amplitude A
- The phase offset δ for periodic pulses
- The amplitude offset c .

By arranging each parameter in an array a sequence is defined. An example of a possible input structure array `sequenceData(1)` is

```
% Set channel 1
% Define sequence details
sequenceData(1).channel = 1;
sequenceData(1).type = {'const', 'sin', 'const'};
sequenceData(1).amplitude = {-5, 10, -5};
sequenceData(1).offset = {0, 0, 0};
sequenceData(1).phase = {0, pi/2, 0};
sequenceData(1).frequency = {10, 1, 10};
sequenceData(1).duration = {10, 5, 10};
```

With these parameters a sequence is defined. In our experiment we write the whole sequence into the memory of the AWG and after a trigger signal all segments are successively called. If we need a time period where no pulse is applied, we program a constant segment with a voltage value of zero. These constant segments will play an extra role in the following calculations as they are only determined by a minimal length.

To program and calculate a sequence we loop over each sequence constituents, i.e. the different entries (e.g. “type”) in the input array.

B.1 Sample rate

We start with determining the sample rate. The sample rate is a crucial part of all coming calculations as it sets the timing and accuracy of the sequence.

Further, if the frequency of a constituent and the sample rate do not match, i.e. they do not have a common divisor, each repeated segment introduces a small time error. In this case a period, e.g. of a sine, cannot be represented by an integer number of samples. For example let the frequency of the sine be $f = 15$ MHz and the sample rate 100 MSa/s. From the frequency of the sine we infer a duration of one period of $\tau = 1/15 \mu\text{s} \approx 0.067 \mu\text{s}$. As the time difference between two consecutive samples is $0.01 \mu\text{s}$ the last sample of the period will appear after $0.06 \mu\text{s}$. However, the next sample, introducing the beginning of the next period, will not appear after $0.067 \mu\text{s}$ but after $0.07 \mu\text{s}$. Thus, a time error of $\approx 0.003 \mu\text{s}$ is introduced with every repetition. By repeating this segment, even a small error can become significant. To avoid this behavior it is necessary to find a sample rate, that matches the frequency of all employed pulses. For this we calculate the least common multiple of all occurring frequencies, i.e. $\text{lcm}\{f_i\}, i = 1, 2, 3, \dots$. Here, we ignore parts of the signal where the output is set to a constant voltage. Further, we have to consider, that the least common multiple is only well defined for integers. Thus, before calculating it, we determine the number of decimal places p_i for each constituent i . Alternatively we fix the number of decimal places p_{fixed} that should be considered in the calculations $p_i = p_{\text{fixed}}, \forall i$.

```

% Calculate decimal places
for cnt = 1:length(frequency)
NrDigits = [NrDigits, numel(regexprep( ...
... num2str(abs(frequency(cnt))), '\d*\D', ''))];
end

% Or fix number of decimal places that are considered
Precision = 6;
NrDigits = ones(1, length(frequency))*Precision;

```

We then generate integer frequencies via $f_{i,\text{round}} \approx f_i \times 10^{p_i}$, where we round on integers. With these integer frequency values we calculate the least common multiple $m = \text{lcm}\{f_{i,\text{round}}\}$. To account for the multiplication with 10_i^p due to the number of decimal places we determine $\tilde{m} = m \times 10^{-\min_i\{p_i\}}$. Then $\tilde{m}$ gives us the least common multiple from which our sample rate is determined via

$$f_{\text{Sample}} = \left\lfloor \frac{f_{\text{Sample, max}}}{\tilde{m}} \right\rfloor \tilde{m}, \quad (\text{B.1})$$

where $f_{\text{Sample, max}}$ is the maximum possible sample rate of the device and $\lfloor \cdot \rfloor$ denotes the floor function. This calculation ensures that the sample rate is as high as possible.

B.2 Replay mode

With the best fitting sample rate we can continue to determine the repetitions needed for each segment to reconstitute the pulse. Here, we have to consider two limits. First, the maximum number of possible repetitions is limited to 10^6 . Second, a single segment has to constitute of at least 32 Samples. The main idea is to write as little as possible into the memory and make use of the replay mode. Thus we start by checking if our segment has to contain more than one period. To cover both cases of long pulse durations and short periods we calculate for both the ratio

$$\begin{aligned} n_{\text{Duration}} &= \left\lceil \frac{\tau \cdot f}{10^6} \right\rceil \\ n_{\text{Sample}} &= \left\lceil \frac{32}{f_{\text{Sample}}/f} \right\rceil. \end{aligned} \quad (\text{B.2})$$

n_{Duration} and n_{Sample} are positive integers and give the number of periods based on the maximum repetitions and the minimum number of samples, respectively. Here, f denotes the frequency of the signal and f_{Sample} the sample rate. τ is the duration of the pulse and $\lceil \cdot \rceil$ is the ceiling function. Thus, by taking the maximum of both we get the number of periods that make up a segment $n_{\text{P}} = \max\{n_{\text{Duration}}, n_{\text{Sample}}\}$. With this number, we can directly calculate the numbers of repetitions of the signal,

that fit into the given duration by

$$R = \left\lfloor \frac{\tau \cdot f}{n_P} \right\rfloor . \quad (\text{B.3})$$

Usually the duration of the pulse does not match a multiple of the time of one period. Thus, the remaining part is determined by calculating the part of the full signal, that has to be added once at the end to match the given duration

$$T = \left(\tau - R \frac{n_P}{f} \right) \cdot \frac{f}{n_P} . \quad (\text{B.4})$$

B.3 Definition of functions

To program our pulses we send a data array to the device. To calculate this array we start with defining a linear time array $\mathbf{T}$, which has a time spacing of $1/f_{\text{Sample}}$ and the amount of points $\tau \cdot f$, where τ is the duration of the segment. With the internal `sin` function in MATLAB we can calculate a sine via

$$\mathbf{D} = \frac{A}{2} \sin(2\pi f \mathbf{T} + \delta) + c , \quad (\text{B.5})$$

where, A is the given peak-peak amplitude, f the frequency, δ a phase offset and c a voltage offset. These values are given by the user. To extract the correct high and low voltage limits one has to additionally scan the data array for its minimum and maximum value, that are then given to the AWG. We use this scheme to calculate our periodic pulses.

Besides employing only a single sine for transfer pulses we use multiple overlapping sines with different frequencies to form for example a box-like potential, cf. section 3.4. To create the data array we calculate the sum of N different sine by

$$\mathbf{D} = \sum_i^N \frac{A_i}{2} \sin(2\pi f_i \mathbf{T} + \delta_i) + c_i , \quad (\text{B.6})$$

where the index i denotes the individual sine. An additional obstacle is to find the correct duration of one period. As several different frequencies are applied, a common multiple of those has to be found to determine the time of one period.

B.4 Adaption to our experiment

In the previous paragraphs, we discussed how we program a signal that we can send to the AWG. A crucial point to make the single segments as short as possible was to find a suiting sample rate for all occurring frequencies. However, sometimes it is not easy to find a sample rate that suits all frequencies in a sequence. Further, the sequences have a duration up to a few seconds. In section 4 we discuss the readout scheme and we see that we need the AWG at the beginning for the state

preparation and at the end to split the population. While both parts (beginning, splitting) consist of a few pulses in a row, during the mw dressing the AWG is set to zero for up to ≈ 1 s. Using these long parts, where the output is set to zero we program the shortest possible constant segment and loop over it. For the pulses however, we do not use the replay mode and program each pulse into a single segment instead. With this, we can make use of the high sample rate and thus the high accuracy as well as the fast switching times of the AWG, without running into memory constraints as the duration of the pulses is rather short.

C Detection of spin-nematic squeezing and entanglement

C.1 Variance analysis

Here, we want to give a short motivation to understand the normalization we choose in the variance analysis for our operator $\hat{F}(\phi)$

$$\begin{aligned}
\Delta^2 F(\phi)_{CS} &= \Delta^2 (\cos(\phi)S_x + \sin(\phi)Q_{yz}) \\
&\stackrel{(4.2)}{=} \cos^2(\phi)\Delta^2 N_{F=1}^- + \sin^2(\phi)\Delta^2 \left(\frac{2}{\sqrt{3}}N_{F=2}^- \right) \\
&\stackrel{(2.8)}{=} \cos^2(\phi) \langle N_{F=1}^- \rangle + \sin^2(\phi) \frac{4}{3} \langle N_{F=2}^- \rangle \\
&\stackrel{(4.2)}{=} \left\langle \cos^2(\phi)S_x + \frac{2}{\sqrt{3}}\sin^2(\phi)Q_{yz} \right\rangle \\
&= \langle N(\phi) \rangle_{CS}
\end{aligned} \tag{C.1}$$

We used that the measurement statistics of N^- for a coherent state can be described by a binomial distribution, which approaches a Gaussian distribution for large N with a variance $\Delta^2 N^- = N$. Further, we applied the rules for variances $\Delta^2(aN) = a^2\Delta^2 N$ and the connection between the observables and the atomic densities.

C.2 Jackknife method

To estimate the uncertainty for the variance analysis, we use the **jackknife** method from MATLAB, which is based on the idea of bootstrap sampling [48]. In the following we will give a short introduction of the calculation. A review of the topic is given in [36].

From our data set, we compute the variance using all the experimental realizations. Thus, we do not have independent measurement sets to determine the uncertainty of the data set. To overcome this limitation, we use the jackknife method, which is based on resampling. From our n data points it takes a subset of size $n - 1$ and calculates the variance. This is repeated n times until every possible subset has been used. This yields a distribution of the variances and allows to estimate an uncertainty from it.

References

- [1] Martin B Plenio and Shashank S Virmani. *An Introduction to Entanglement Theory*, pages 173–209. Springer International Publishing, Cham, 2014. ISBN 978-3-319-04063-9. doi:10.1007/978-3-319-04063-9_8. URL https://doi.org/10.1007/978-3-319-04063-9_8.
- [2] Nicolai Friis, Giuseppe Vitagliano, Mehul Malik, and Marcus Huber. Entanglement certification from theory to experiment. *Nature Reviews Physics*, 1(1):72–87, jan 2019. ISSN 2522-5820. doi:10.1038/s42254-018-0003-5. URL <http://www.nature.com/articles/s42254-018-0003-5>.
- [3] Y. S. Li, B. Zeng, X. S. Liu, and G. L. Long. Entanglement in a two-identical-particle system. *Physical Review A - Atomic, Molecular, and Optical Physics*, 64(5), 2001. ISSN 10941622. doi:10.1103/PhysRevA.64.054302.
- [4] Daniel Cavalcanti. Split, but still attached. *Science*, 360(6387):376 LP – 377, apr 2018. doi:10.1126/science.aat4590. URL <http://science.sciencemag.org/content/360/6387/376.abstract>.
- [5] Ryszard Horodecki, Paweł Horodecki, Michał Horodecki, and Karol Horodecki. Quantum entanglement. *Reviews of Modern Physics*, 81(2):865–942, jun 2009. ISSN 0034-6861. doi:10.1103/RevModPhys.81.865. URL <https://link.aps.org/doi/10.1103/RevModPhys.81.865>.
- [6] Otfried Gühne and Géza Tóth. Entanglement detection. *Physics Reports*, 474(1-6):1–75, apr 2009. ISSN 03701573. doi:10.1016/j.physrep.2009.02.004. URL <https://linkinghub.elsevier.com/retrieve/pii/S0370157309000623>.
- [7] K. Eckert, J. Schliemann, D. Bruß, and M. Lewenstein. Quantum Correlations in Systems of Indistinguishable Particles. *Annals of Physics*, 299(1):88–127, jul 2002. ISSN 00034916. doi:10.1006/aphy.2002.6268. URL <https://linkinghub.elsevier.com/retrieve/pii/S0003491602962688>.
- [8] Tzu Chieh Wei. Exchange symmetry and global entanglement and full separability. *Physical Review A - Atomic, Molecular, and Optical Physics*, 81(5):054102, may 2010. ISSN 10502947. doi:10.1103/PhysRevA.81.054102. URL <https://link.aps.org/doi/10.1103/PhysRevA.81.054102>.
- [9] N. Killoran, M. Cramer, and M. B. Plenio. Extracting Entanglement from Identical Particles. *Physical Review Letters*, 112(15):150501, apr 2014. ISSN 0031-9007. doi:10.1103/PhysRevLett.112.150501. URL <https://link.aps.org/doi/10.1103/PhysRevLett.112.150501>.
- [10] Jian Ma, Xiaoguang Wang, C. P. Sun, and Franco Nori. Quantum spin squeezing. nov 2010. doi:10.1016/j.physrep.2011.08.003. URL <http://arxiv.org/abs/1011.2978><http://dx.doi.org/10.1016/j.physrep.2011.08.003>.

- [11] Luca Pezzè, Augusto Smerzi, Markus K. Oberthaler, Roman Schmied, and Philipp Treutlein. Quantum metrology with nonclassical states of atomic ensembles. *Reviews of Modern Physics*, 90(3):035005, sep 2018. ISSN 0034-6861. doi:10.1103/RevModPhys.90.035005. URL <https://link.aps.org/doi/10.1103/RevModPhys.90.035005>.
- [12] Karsten Lange, Jan Peise, Bernd Lücke, Ilka Kruse, Giuseppe Vitagliano, Iagoba Apellaniz, Matthias Kleinmann, Géza Tóth, and Carsten Klempt. Entanglement between two spatially separated atomic modes. *Science*, 360(6387):416–418, apr 2018. ISSN 0036-8075. doi:10.1126/science.aao2035. URL <https://www.sciencemag.org/lookup/doi/10.1126/science.aao2035>.
- [13] Philipp Kunkel, Maximilian Prüfer, Helmut Strobel, Daniel Linnemann, Anika Frölian, Thomas Gasenzer, Martin Gärttner, and Markus K. Oberthaler. Spatially distributed multipartite entanglement enables EPR steering of atomic clouds. *Science*, 360(6387):413–416, apr 2018. ISSN 0036-8075. doi:10.1126/science.aao2254. URL <https://www.sciencemag.org/lookup/doi/10.1126/science.aao2254>.
- [14] Philipp Kunkel. *Splitting a Bose Einstein condensate enables EPR steering and simultaneous readout of noncommuting observables*. PhD thesis, Universität Heidelberg, 2019.
- [15] Wolfgang Muessel. *Scalable Spin Squeezing for Quantum-Enhanced Magnetometry with Bose-Einstein Condensates*. PhD thesis, Universität Heidelberg, 2014.
- [16] Wolfgang Muessel, Helmut Strobel, Maxime Joos, Eike Nicklas, Ion Stroescu, Jiří Tomkovič, David B. Hume, and Markus K. Oberthaler. Optimized absorption imaging of mesoscopic atomic clouds. *Applied Physics B*, 113(1):69–73, oct 2013. ISSN 0946-2171. doi:10.1007/s00340-013-5553-8. URL <http://link.springer.com/10.1007/s00340-013-5553-8>.
- [17] Helmut Strobel, Wolfgang Muessel, Daniel Linnemann, Tilman Zibold, David B. Hume, L. Pezze, Augusto Smerzi, and Markus K. Oberthaler. Fisher information and entanglement of non-Gaussian spin states. *Science*, 345(6195):424–427, jul 2014. ISSN 0036-8075. doi:10.1126/science.1250147. URL <https://www.sciencemag.org/lookup/doi/10.1126/science.1250147>.
- [18] Anika Frölian. Implementation of local addressability in a bose-einstein condensate. Master’s thesis, Universität Heidelberg, 2017.
- [19] Stefan Lannig. Experimental Control of a Spin-1 Bose-Einstein Condensate. Master’s thesis, Universität Heidelberg, 2018.
- [20] J. J. Sakurai and Richard L. Liboff. Modern Quantum Mechanics. *American Journal of Physics*, 54(7):668–668, jul 1986. ISSN 0002-9505.

- doi:10.1119/1.14491. URL <http://aapt.scitation.org/doi/10.1119/1.14491>.
- [21] C. D. Hamley, C. S. Gerving, T. M. Hoang, E. M. Bookjans, and M. S. Chapman. Spin-Nematic Squeezed Vacuum in a Quantum Gas. *Nature Physics*, 8(4):305–308, nov 2011. ISSN 1745-2473. doi:10.1038/nphys2245. URL <http://www.nature.com/articles/nphys2245><http://arxiv.org/abs/1111.1694><http://dx.doi.org/10.1038/nphys2245>.
- [22] Yaomin Di, Yan Wang, and Hairui Wei. Dipole–quadrupole decomposition of two coupled spin 1 systems. *Journal of Physics A: Mathematical and Theoretical*, 43(6):065303, feb 2010. ISSN 1751-8113. doi:10.1088/1751-8113/43/6/065303. URL <https://iopscience.iop.org/article/10.1088/1751-8113/43/6/065303>.
- [23] P. Jordan. Der Zusammenhang der symmetrischen und linearen Gruppen und das Mehrkörperproblem. *Zeitschrift für Physik*, 94(7-8):531–535, jul 1935. ISSN 1434-6001. doi:10.1007/BF01330618. URL <http://link.springer.com/10.1007/BF01330618>.
- [24] Yuki Kawaguchi and Masahito Ueda. Spinor Bose–Einstein condensates. *Physics Reports*, 520(5):253–381, nov 2012. ISSN 03701573. doi:10.1016/j.physrep.2012.07.005. URL <https://linkinghub.elsevier.com/retrieve/pii/S0370157312002098>.
- [25] J Schwinger. On Angular Momentum. jan 1952. doi:10.2172/4389568. URL <https://www.osti.gov/biblio/4389568><http://www.osti.gov/servlets/purl/4389568-jZqrUJ/native/>.
- [26] C. K. Law, H. Pu, and N. P. Bigelow. Quantum Spins Mixing in Spinor Bose-Einstein Condensates. *Physical Review Letters*, 81(24):5257–5261, dec 1998. ISSN 0031-9007. doi:10.1103/PhysRevLett.81.5257. URL <https://link.aps.org/doi/10.1103/PhysRevLett.81.5257>.
- [27] L.-M. Duan, J. I. Cirac, and P. Zoller. Quantum entanglement in spinor Bose-Einstein condensates. *Physical Review A*, 65(3):033619, feb 2002. ISSN 1050-2947. doi:10.1103/PhysRevA.65.033619. URL <https://link.aps.org/doi/10.1103/PhysRevA.65.033619>.
- [28] Alice Sinatra, Carlos Lobo, and Yvan Castin. The truncated Wigner method for Bose-condensed gases: limits of validity and applications. *Journal of Physics B: Atomic, Molecular and Optical Physics*, 35(17):3599–3631, sep 2002. ISSN 0953-4075. doi:10.1088/0953-4075/35/17/301. URL <http://arxiv.org/abs/cond-mat/0201217><http://dx.doi.org/10.1088/0953-4075/35/17/301><https://iopscience.iop.org/article/10.1088/0953-4075/35/17/301>.

- [29] Matthias Bartelmann, Björn Feuerbacher, Timm Krüger, Dieter Lüst, Anton Rebhan, and Andreas Wipf. *Theoretische Physik 3 — Quantenmechanik*. Springer Berlin Heidelberg, Berlin, Heidelberg, 2018. ISBN 978-3-662-56071-6. doi:10.1007/978-3-662-56072-3. URL <http://link.springer.com/10.1007/978-3-662-56072-3>.
- [30] Masahiro Kitagawa and Masahito Ueda. Squeezed spin states. *Physical Review A*, 47(6):5138–5143, jun 1993. ISSN 1050-2947. doi:10.1103/PhysRevA.47.5138. URL <https://link.aps.org/doi/10.1103/PhysRevA.47.5138>.
- [31] H. P. Robertson. The Uncertainty Principle. *Physical Review*, 34(1):163–164, jul 1929. ISSN 0031-899X. doi:10.1103/PhysRev.34.163. URL <https://link.aps.org/doi/10.1103/PhysRev.34.163>.
- [32] C. F. Ockeloen, A. F. Tauschinsky, R. J. C. Spreeuw, and S. Whitlock. Detection of small atom numbers through image processing. *Physical Review A*, 82(6):061606, dec 2010. ISSN 1050-2947. doi:10.1103/PhysRevA.82.061606. URL <https://link.aps.org/doi/10.1103/PhysRevA.82.061606>.
- [33] Helmut Strobel. *Fisher Information and entanglement of non-Gaussian spin states*. PhD thesis, Universität Heidelberg, 2016.
- [34] Fabrice Gerbier, Artur Widera, Simon Fölling, Olaf Mandel, and Immanuel Bloch. Resonant control of spin dynamics in ultracold quantum gases by microwave dressing. *Physical Review A*, 73(4):041602, apr 2006. ISSN 1050-2947. doi:10.1103/PhysRevA.73.041602. URL <https://link.aps.org/doi/10.1103/PhysRevA.73.041602>.
- [35] Philipp Kunkel, Maximilian Prüfer, Stefan Lannig, Rodrigo Rosa-Medina, Alexis Bonnin, Martin Gärttner, Helmut Strobel, and Markus K. Oberthaler. Simultaneous Readout of Noncommuting Collective Spin Observables beyond the Standard Quantum Limit. *Physical Review Letters*, 123(6):063603, aug 2019. ISSN 0031-9007. doi:10.1103/PhysRevLett.123.063603. URL <https://link.aps.org/doi/10.1103/PhysRevLett.123.063603>.
- [36] Rupert G. Miller. The jackknife-a review. *Biometrika*, 61(1):1–15, 1974. ISSN 00063444. doi:10.1093/biomet/61.1.1. URL <https://academic.oup.com/biomet/article-lookup/doi/10.1093/biomet/61.1.1>.
- [37] Lu-Ming Duan, G. Giedke, J. I. Cirac, and P. Zoller. Inseparability Criterion for Continuous Variable Systems. *Physical Review Letters*, 84(12):2722–2725, mar 2000. ISSN 0031-9007. doi:10.1103/PhysRevLett.84.2722. URL <https://link.aps.org/doi/10.1103/PhysRevLett.84.2722>.
- [38] E. Arthurs and J. L. Kelly. On the Simultaneous Measurement of a Pair of Conjugate Observables. *Bell System Technical Journal*, 44(4):725–729,

- apr 1965. ISSN 00058580. doi:10.1002/j.1538-7305.1965.tb01684.x. URL <https://ieeexplore.ieee.org/document/6769581>.
- [39] Philipp Kunkel, Maximilian Prüfer, Stefan Lannig, Robin Strohmaier, Martin Gärttner, Helmut Strobel, and Markus K. Oberthaler. Experimental detection of entanglement structure in quantum fields. in preparation.
- [40] Alfred Wehrl. On the relation between classical and quantum-mechanical entropy. *Reports on Mathematical Physics*, 16(3):353–358, dec 1979. ISSN 00344877. doi:10.1016/0034-4877(79)90070-3. URL <https://linkinghub.elsevier.com/retrieve/pii/0034487779900703>.
- [41] Kôdi Husimi. Some Formal Properties of the Density Matrix. *Proceedings of the Physico-Mathematical Society of Japan. 3rd Series*, 22(4):264–314, 1940. doi:10.11429/ppmsj1919.22.4.264.
- [42] H. Häffner, C. F. Roos, and R. Blatt. Quantum computing with trapped ions, dec 2008. ISSN 03701573. URL <https://linkinghub.elsevier.com/retrieve/pii/S0370157308003463>.
- [43] G. Wendin. Quantum information processing with superconducting circuits: A review, oct 2017. ISSN 00344885. URL <https://iopscience.iop.org/article/10.1088/1361-6633/aa7e1a>.
- [44] Andrew Steane. Quantum computing. *Reports on Progress in Physics*, 61(2): 117–173, feb 1998. ISSN 0034-4885. doi:10.1088/0034-4885/61/2/002. URL <https://doi.org/10.1088/0034-4885/61/2/002><https://iopscience.iop.org/article/10.1088/0034-4885/61/2/002>.
- [45] A. Sørensen, L.-M. Duan, J. I. Cirac, and P. Zoller. Many-particle entanglement with Bose–Einstein condensates. *Nature*, 409(6816):63–66, jan 2001. ISSN 0028-0836. doi:10.1038/35051038. URL <http://www.nature.com/articles/35051038>.
- [46] E. Farhi, J. Goldstone, S. Gutmann, J. Lapan, A. Lundgren, and D. Preda. A Quantum Adiabatic Evolution Algorithm Applied to Random Instances of an NP-Complete Problem. *Science*, 292(5516):472–475, apr 2001. ISSN 0036-8075. doi:10.1126/science.1057726. URL <https://www.sciencemag.org/lookup/doi/10.1126/science.1057726>.
- [47] Robert Raussendorf and Hans J. Briegel. A One-Way Quantum Computer. *Physical Review Letters*, 86(22):5188–5191, may 2001. ISSN 0031-9007. doi:10.1103/PhysRevLett.86.5188. URL <https://link.aps.org/doi/10.1103/PhysRevLett.86.5188>.

- [48] B. Efron. Bootstrap Methods: Another Look at the Jackknife. *The Annals of Statistics*, 7(1), jan 1979. ISSN 0090-5364. doi:10.1214/aos/1176344552. URL <https://projecteuclid.org/journals/annals-of-statistics/volume-7/issue-1/Bootstrap-Methods-Another-Look-at-the-Jackknife/10.1214/aos/1176344552.full>.

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Erklärung:

Ich versichere, dass ich diese Arbeit selbstständig verfasst habe und keine anderen als die angegebenen Quellen und Hilfsmittel benutzt habe.

Heidelberg, den 30.04.2021

Robin Strohmaier
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