

## A<sub>2</sub>MnXO<sub>4</sub> Family (A = Li, Na, Ag; X = Si, Ge): Structural and Magnetic Properties

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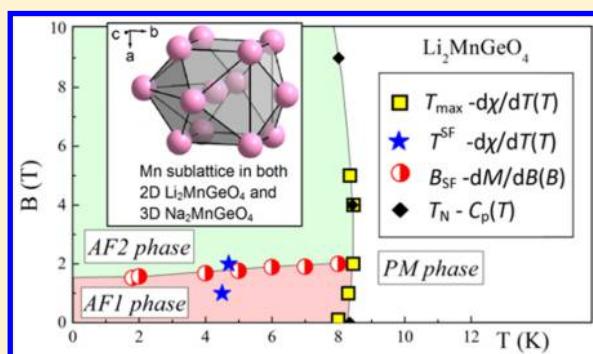
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### Supporting Information

**ABSTRACT:** Four new manganese germanates and silicates, A<sub>2</sub>MnGeO<sub>4</sub> (A = Li, Na) and A<sub>2</sub>MnSiO<sub>4</sub> (A = Na, Ag), were prepared, and their crystal structures were determined using the X-ray Rietveld method. All of them contain all components in tetrahedral coordination. Li<sub>2</sub>MnGeO<sub>4</sub> is orthorhombic (*Pmn*2<sub>1</sub>) layered, isostructural with Li<sub>2</sub>CdGeO<sub>4</sub>, and the three other compounds are monoclinic (*Pn*) cristobalite-related frameworks. As in other stuffed cristobalites of various symmetry (*Pn* A<sub>2</sub>MXO<sub>4</sub>, *Pna*2<sub>1</sub> and *Pbca* AMO<sub>2</sub>), average bond angles on bridging oxygens (here, Mn–O–X) increase with increasing A/X and/or A/M radius ratios, indicating the trend to the ideal cubic (*Fd*3*m*) structure typified by CsAlO<sub>2</sub>. The sublattices of the magnetic Mn<sup>2+</sup> ions in both structure types under study (*Pmn*2<sub>1</sub> and *Pn*) are essentially the same; namely, they are pseudocubic eutaxy with 12 nearest neighbors. The magnetic properties of the four new phases plus Li<sub>2</sub>MnSiO<sub>4</sub> were characterized by carrying out magnetic susceptibility, specific heat, magnetization, and electron spin resonance measurements and also by performing energy-mapping analysis to evaluate their spin exchange constants. Ag<sub>2</sub>MnSiO<sub>4</sub> remains paramagnetic down to 2 K, but A<sub>2</sub>MnXO<sub>4</sub> (A = Li, Na; X = Si, Ge) undergo a three-dimensional antiferromagnetic ordering. All five phases exhibit short-range AFM ordering correlations, hence showing them to be low-dimensional magnets and a magnetic field induced spin-reorientation transition at *T* < *T<sub>N</sub>* for all AFM phases. We constructed the magnetic phase diagrams for A<sub>2</sub>MnXO<sub>4</sub> (A = Li, Na; X = Si, Ge) on the basis of the thermodynamic data in magnetic fields up to 9 T. The magnetic properties of all five phases experimentally determined are well explained by their spin exchange constants evaluated by performing energy-mapping analysis.



## 1. INTRODUCTION

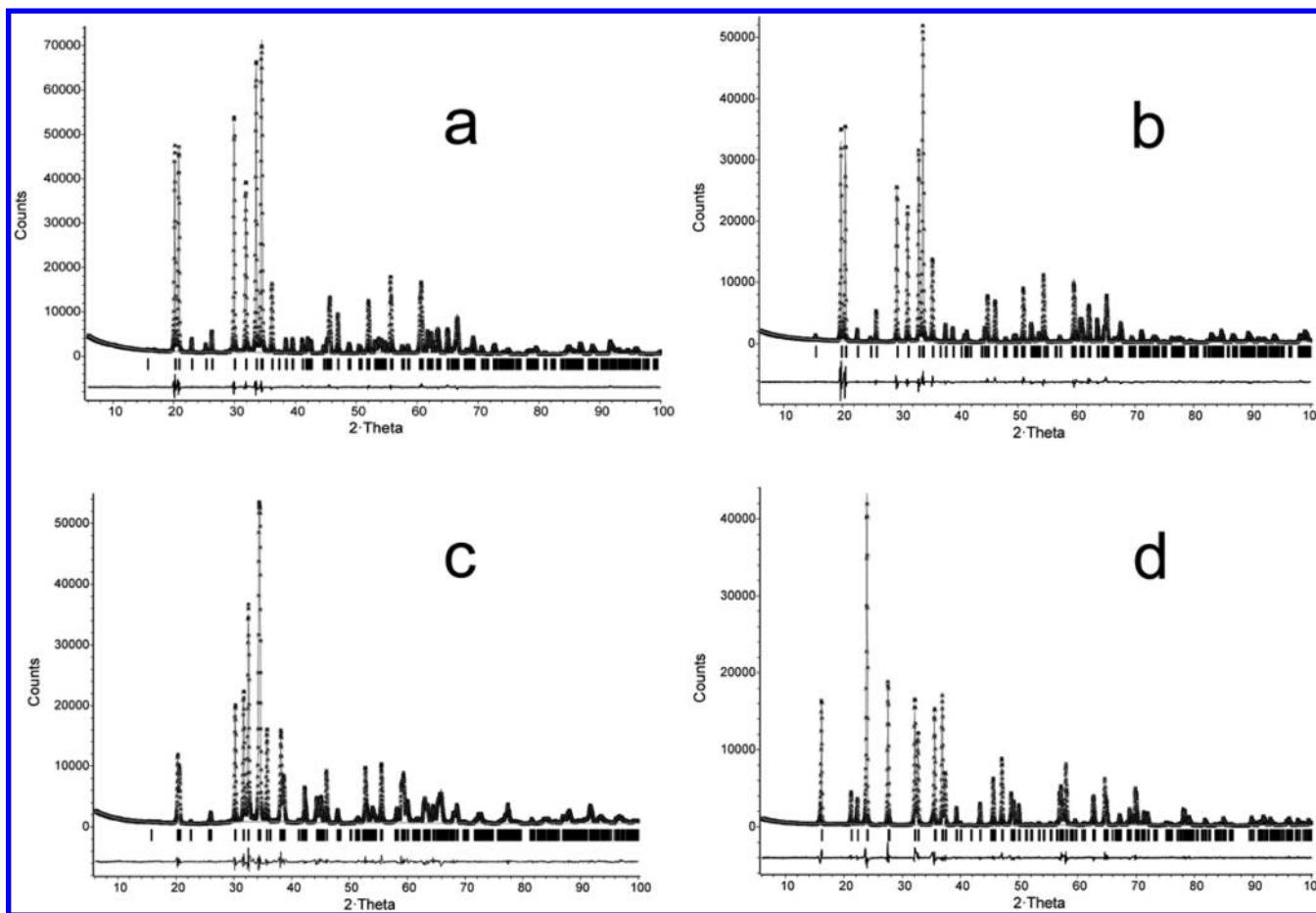
Compounds of the A<sub>2</sub>MXO<sub>4</sub> family (A = Li, Na, Ag; M = Be, Mg, Mn, Fe, Co, Zn, Cd; X = Si, Ge) present a variety of tetrahedral structure types and physical properties. Li<sub>2</sub>MSiO<sub>4</sub> (M = Mn, Fe, Co) have been intensely studied as materials for positive electrodes of lithium-ion batteries.<sup>1–11</sup> Their sodium counterparts are of interest for sodium-ion batteries<sup>12–14</sup> or capacitors.<sup>15</sup> Ag<sub>2</sub>ZnGeO<sub>4</sub> has attracted attention as a photocatalyst<sup>16,17</sup> and Li<sub>2</sub>CoGeO<sub>4</sub> as an oxygen evolution catalyst.<sup>18</sup> Several phases of this family are solid-state Na<sup>+</sup> or Ag<sup>+</sup> cation conductors<sup>19–24</sup> and hosts for luminescent ions.<sup>25–29</sup> The most intensely studied members of this family are based on magnetic cations M<sup>2+</sup> = Mn, Fe, and Co, but surprisingly little is known

about their magnetic structures and properties. Only Na<sub>2</sub>CoXO<sub>4</sub> (X = Si, Ge)<sup>30,31</sup> and Li<sub>2</sub>MSiO<sub>4</sub> (M = Mn, Fe, Co)<sup>3,6,7,11,30,32,33</sup> have been studied in some detail. Five of the six possible Mn<sup>2+</sup> compounds of this family, however, remain unexplored regarding magnetism.

The present work is concerned with the manganese compounds, namely, A<sub>2</sub>MnSiO<sub>4</sub> (A = Li, Na, Ag) and A<sub>2</sub>MnGeO<sub>4</sub> (A = Li, Na). The indexed powder diffraction patterns of these five compounds were reported by us earlier,<sup>34</sup> and the crystal structure of Li<sub>2</sub>MnSiO<sub>4</sub> (*P2*<sub>1</sub>/*n*) was also

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**Figure 1.** Experimental XRD patterns and Rietveld refinement results: (a)  $\text{Na}_2\text{MnSiO}_4$ , (b)  $\text{Na}_2\text{MnGeO}_4$ , (c)  $\text{Ag}_2\text{MnSiO}_4$ , and (d)  $\text{Li}_2\text{MnGeO}_4$ . Stars, experimental points; gray lines, calculated profiles; thick light gray line, background; vertical bars, Bragg positions; bottom lines, difference profiles.

reported.<sup>4,33</sup> There is only a brief mention of the lattice parameters for  $\text{Li}_2\text{MnGeO}_4$  ( $Pmn2_1$ ) without any structural detail,<sup>35</sup> and a density functional theory (DFT) study on  $\text{Na}_2\text{MnSiO}_4$  ( $Pn$ ) with diffraction data only used for lattice parameter refinement.<sup>10</sup> Here, we report the preparation, crystal structures, and magnetic properties of  $\text{A}_2\text{MnSiO}_4$  ( $\text{A} = \text{Na, Ag}$ ) and  $\text{A}_2\text{MnGeO}_4$  ( $\text{A} = \text{Li, Na}$ ) as well as a more careful study of the magnetism of  $\text{Li}_2\text{MnSiO}_4$ . We evaluate the spin exchange interactions of these five compounds by performing the energy-mapping analysis<sup>36,37</sup> based on DFT calculations and discuss their magnetic properties.

## 2. EXPERIMENTAL SECTION

Starting materials were reagent-grade alkali carbonates, manganese oxide, germanium oxide, zinc oxide, cadmium oxide, hydrous silica, and silver nitrate. The carbonates and  $\text{GeO}_2$  were dried at 420 K, and  $\text{CdO}$  was calcined at 900 K.  $\text{SiO}_2 \cdot x\text{H}_2\text{O}$  and  $\text{MnO}_y$  were analyzed by weight change on calcinations at 1270 K (to give  $\text{SiO}_2$ ) and 1020 K (to give phase-pure  $\text{Mn}_2\text{O}_3$ ), respectively, and then used in their pristine forms.  $\text{A}_2\text{MnXO}_4$  ( $\text{A} = \text{Li, Na}$ ) were obtained from carefully mixed stoichiometric amounts of the reagents by conventional solid-state reactions in flowing hydrogen ( $\text{X} = \text{Si}$ ) or nitrogen ( $\text{X} = \text{Ge}$ ) with intermittent grindings and pressings. The temperatures of the final heat treatments were 1250 K for both germanates, 1220 K for  $\text{Na}_2\text{MnSiO}_4$ , and 1320 K for  $\text{Li}_2\text{MnSiO}_4$ . The diamagnetic analogues  $\text{Li}_2\text{CdGeO}_4$ <sup>38</sup> and  $\text{Na}_2\text{ZnSiO}_4$ <sup>39,40</sup> were prepared similarly in air at 1170 K.  $\text{Ag}_2\text{MnSiO}_4$  was prepared from  $\text{Na}_2\text{MnSiO}_4$  by ion exchange in slight excess of molten  $\text{AgNO}_3$  for 2 h at 570 K, which was followed

by washing with distilled water and drying. The silver content was determined by thiocyanate titration after digestion in hot  $\text{H}_2\text{SO}_4$  and dilution. The phase purity of all prepared compounds was verified by powder X-ray diffraction (XRD). Scans for the structure analysis were performed with  $\text{Cu K}\alpha$  radiation using a rotating-anode D/max-RC diffractometer (Rigaku, Japan) equipped with a secondary beam graphite monochromator. For Rietveld refinements, the GSAS +EXPGUI suite was used.<sup>41,42</sup>

The magnetic properties were measured using a Quantum Design SQUID XL-5 magnetometer. The temperature dependence of magnetic susceptibility  $\chi = M/B$  was measured in the temperature range 1.8–300 K at the magnetic field  $B = 0.1$  T and in the  $T$  range 2–20 K in various applied fields  $B \leq 5$  T. The isothermal magnetization curves  $M$  vs  $B$  were recorded for magnetic fields  $B \leq 5$  T at various temperatures  $T < T_N$  after cooling the sample in zero field. The specific heat measurements were carried out by a relaxation method using a Quantum Design PPMS system. The plate-shaped samples of  $\text{Li}_2\text{MnGeO}_4$ ,  $\text{Li}_2\text{CdGeO}_4$ ,  $\text{Na}_2\text{MnSiO}_4$ ,  $\text{Na}_2\text{ZnSiO}_4$ , and  $\text{Li}_2\text{MnSiO}_4$  of ~0.2 mm thickness and 4–6 mg mass used for the experiments were obtained by cold pressing of the polycrystalline powder. The data were collected at zero magnetic field and under applied fields up to 9 T in the temperature range 1.9–280 K. Electron spin resonance (ESR) studies were carried out using an X-band ESR spectrometer CMS 8400 (ADANI) ( $f \approx 9.4$  GHz,  $B \leq 0.7$  T) equipped with a low-temperature mount operating in the range  $T = 7$ –300 K. The effective  $g$ -factors of our samples were calculated with respect to an external reference for the resonance field using BDPA (a,g-bisphenyl-*b*-phenylallyl) as the reference material, for which  $g_{\text{ref}} = 2.00359$ .

Table 1. XRD Refinement Details for  $A_2MnXO_4$ 

	$Li_2MnGeO_4$	$Na_2MnSiO_4$	$Na_2MnGeO_4$	$Ag_2MnSiO_4$
formula weight	205.4	193.0	237.5	362.7
crystal system	orthorhombic	monoclinic	monoclinic	monoclinic
space group	$Pmn2_1$	$P1n1$	$P1n1$	$P1n1$
temperature of data collection (K)	301	307	301	298
$a$ (Å)	6.46751(7)	7.02857(7)	7.16427(8)	6.93888(8)
$b$ (Å)	5.47639(5)	5.60957(5)	5.72970(7)	5.64791(7)
$c$ (Å)	5.05380(5)	5.33391(5)	5.40710(6)	5.50969(7)
$\beta$ (deg)	90	89.7949(13)	90.109(2)	90.3942(13)
$V$ (Å <sup>3</sup> )	179.00	210.30	221.96	215.92
$Z$	2	2	2	2
$R_{wp}$	0.0884	0.0726	0.0963	0.0667
$R_{exp}$	0.0326	0.0205	0.0276	0.0226
$R_{F^2}$	0.02991	0.04033	0.05102	0.02826

Table 2. Important Interatomic Distances (Å) and Bond Angles (deg) in  $A_2MnXO_4$ <sup>a</sup>

	$Li_2MnGeO_4$	$Na_2MnGeO_4$	$Na_2MnSiO_4$	$Ag_2MnSiO_4$
X–O				
	1.744(5)	1.687(7)	1.541(4)	1.602(10)
	1.747(5)	1.737(4)	1.555(3)	1.667(13)
	1.765(4)×2	1.838(5)	1.616(3)	1.695(11)
		1.935(4)	1.715(3)	1.695(12)
average	1.76	1.80	1.61	1.66
sum of ionic radii <sup>43</sup>	1.77		1.64	
Mn–O				
	2.053(4)×2	1.840(10)	2.007(5)	1.962(9)
	2.106(6)	1.892(7)	2.079(5)	2.002(12)
	2.156(6)	2.092(7)	2.082(6)	2.016(10)
		2.120 (7)	2.086(4)	2.111(11)
average	2.09	1.99	2.06	2.02
sum of ionic radii <sup>43</sup>	2.04			
A1–O				
	1.869(16)	2.147(4)	2.255(5)	2.319(10)
	1.969(12)	2.259(8)	2.285(3)	2.359(10)
	2.011(9)	2.345(5)	2.476(3)	2.460(11)
	2.021(11)	2.558(5)	2.484(3)	2.469(11)
average	1.97	2.33	2.38	2.40
sum of ionic radii <sup>43</sup>	1.97	2.37		2.38
A2–O				
	2.235(4)	2.166(3)	2.230(12)	
	2.358(10)	2.300(5)	2.342(10)	
	2.397(5)	2.370(3)	2.354(12)	
	2.424(5)	2.386(3)	2.383(11)	
average		2.35	2.31	2.33
sum of ionic radii <sup>43</sup>		2.37		2.38
Mn–Mn				
	4.535 × 4	4.905 × 2	4.858 × 2	4.941 × 2
	5.054 × 2	4.912 × 2	4.872 × 2	4.967 × 2
	5.424 × 4	5.407 × 2	5.334 × 2	5.510 × 2
	5.476 × 2	5.730 × 2	5.610 × 2	5.591 × 2
		5.840 × 2	5.662 × 2	5.615 × 2
		5.846 × 2	5.674 × 2	5.648 × 2
angles: Mn–O1–X	115.6(2)×2	118.4(2)	124.3(2)	130.0(6)
Mn–O2–X	110.1(2)	120.6(2)	125.1(2)	132.2(5)
Mn–O3–X	108.8(2)	128.9(2)	132.5(2)	127.0(6)
Mn–O4–X		133.4(2)	126.9(2)	125.8(6)
average	112.5	125.3	127.2	128.8

<sup>a</sup>Minor Li/Mn mixing in  $Li_2MnGeO_4$  and minor Na substitution for Ag in  $Ag_2MnSiO_4$  are ignored in this table but are indicated in Table S2 of the Supporting Information.

### 3. RESULTS AND DISCUSSION

**3.1. Crystal Structures.** The experimental, calculated, and difference XRD profiles for the four compounds are shown in Figure 1. Details of the XRD refinements are summarized in Table 1; the important interatomic distances and bond angles

are in Table 2, and details of data collection are presented in Table S1. The atomic coordinates and isotropic thermal parameters are listed in Table S2. Table S3 compares lattice parameters of the new phases with those for the previously known analogues.

$\text{Li}_2\text{MnGeO}_4$  is light gray, a typical color of high-spin  $\text{Mn}^{2+}$  ( $d^5$ ) compounds. Its XRD pattern was completely indexed in the orthorhombic system. The sole independent systematic absence is found for the reflections  $h0l$  ( $h + l$  odd absent). This indicates two possible space groups:  $\text{P}m\text{n}m$  and  $\text{P}m\text{n}2_1$ . The overall similarity of the XRD pattern with that for  $\text{Li}_2\text{CdGeO}_4$ <sup>38</sup> suggests the latter group as the most probable. With the crystal structure of  $\text{Li}_2\text{CdGeO}_4$  taken as a starting model, the refinement converged rapidly to reasonable reliability factors. The unit cell volumes correlate with ionic sizes (Table S3 of the Supporting Information); the average bond lengths of  $\text{Li}–\text{O}$  and  $\text{Ge}–\text{O}$  agree well with the corresponding ionic radii sums, but two of the four  $\text{Mn}–\text{O}$  distances are somewhat longer than expected (Table 2). In contrast to the  $\text{Cd}$  prototype, a small degree of  $\text{Li}/\text{Mn}$  mixing was found in this structure. This is so because the ionic radii of  $\text{Mn}^{2+}$  and  $\text{Li}^+$  are similar.

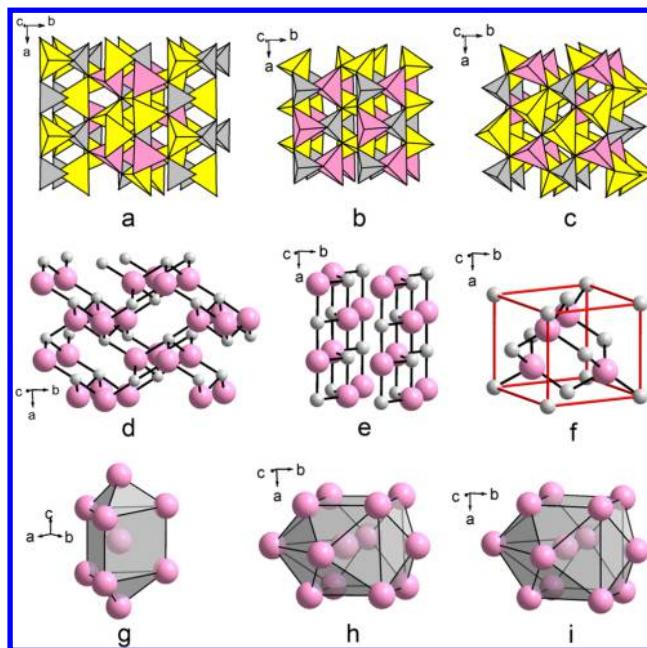
Both  $\text{Na}_2\text{MnSiO}_4$  and  $\text{Na}_2\text{MnGeO}_4$  are light brown. This darkening may be attributed to their partial oxidation in air, which leads to the extraction of sodium, hence forming a mixed-valence state  $\text{Mn}^{2+}/\text{Mn}^{3+}$  on the surface. Indeed, XPS study of  $\text{Na}_2\text{MnSiO}_4$  shows very weak peaks from silicon and manganese but strong peaks from sodium, carbon, and oxygen, indicating that the surface layer consists mostly of  $\text{Na}_2\text{CO}_3$ . The weak response from Mn is in reasonable agreement with that from  $\text{MnO}$ , although admixture of  $\text{Mn}^{3+}$  cannot be excluded. In any case, this is only a surface layer, whereas all bulk properties (ESR, magnetization, bond lengths, etc.) unambiguously correspond to  $\text{Mn}^{2+}$ , as shown below. The associated deviation from the stoichiometry is too small to be detected from the refined Na site occupancies. In general, color changes and ESR are much more sensitive to the presence of mixed valence than most analytical methods.

The powder patterns for both sodium compounds might be indexed in an orthorhombic system with unit cell parameters similar to those for  $\text{Li}_2\text{MnGeO}_4$  (only slightly larger, in accordance with the larger size of  $\text{Na}^+$ , see Table 1), and even systematic absences were the same. However, the accuracy of the fit was much worse compared with the case of  $\text{Li}_2\text{MnGeO}_4$ , especially for the silicate. This indicated that the true symmetry is lower. Moreover, the relative intensities differed drastically, indicating another structure type. We note a strong analogy with the family of monoclinic phases having space group  $\text{P}1n1$  with angle  $\beta$  very close to  $90^\circ$ , typified by  $\text{Na}_2\text{ZnXO}_4$  ( $\text{X} = \text{Si}, \text{Ge}$ ).<sup>39,40</sup> This nonstandard setting for the space group 7 is preferred to the standard one,  $\text{P}1c1$ , to make the pseudo-orthorhombic metrics evident and facilitate the comparison with truly orthorhombic counterparts. The same holds for  $\text{Li}_2\text{MnSiO}_4$  ( $\text{P}12_1/n$  instead of  $\text{P}12_1/c1$ ). Thus, the space group for  $\text{Na}_2\text{MnGeO}_4$  in the PDF card 00-56-94 should be changed from  $\text{P}mn2_1$  to  $\text{P}1n1$ . The XRD pattern of  $\text{Ag}_2\text{MnSiO}_4$  (PDF card 00-55-699) was initially indexed with doubled  $c$ -axis, but this resulted only from a typo in one  $2\Theta$  value. After correcting the typo, the pattern was completely indexed with a small unit cell analogous to that of the sodium precursor.

Taking the  $\text{Zn}$  analogues for the starting models, the crystal structures of  $\text{Na}_2\text{MnXO}_4$  ( $\text{X} = \text{Si}, \text{Ge}$ ) and  $\text{Ag}_2\text{MnSiO}_4$  were successfully refined. The unit cell volumes show reasonable correlations with the ionic radii (Table S3 of the Supporting Information). Differences between the average bond lengths and the corresponding sums of ionic radii do not exceed 0.06 Å in all cases (Table 2). In contrast to  $\text{Li}_2\text{MnGeO}_4$ , no  $\text{A}/\text{Mn}$  mixing was found, which is due to the larger differences in ionic radii. In  $\text{Ag}_2\text{MnSiO}_4$ , however, our analytical data indicated

somewhat reduced Ag content (found, 57.7 wt %; calcd 59.5 wt %) due to an incomplete ion exchange. Therefore, mixed  $\text{Ag}/\text{Na}$  occupancy was refined in the two independent silver sites and resulted in Na fractions of  $\sim 0.02$  and  $\sim 0.07$  (Table S2 of the Supporting Information). Thus, the true formula should be  $\text{Ag}_{1.91}\text{Na}_{0.09}\text{MnSiO}_4$  with 58.0 wt % Ag, in reasonable agreement with the titration data. This minor Na impurity will be ignored for simplicity in our further discussion.

**3.2. Crystal–Chemistry Relations between the Tetrahedral Structures.** The structural results of our work on  $\text{A}_2\text{MnGeO}_4$  ( $\text{A} = \text{Li}, \text{Na}$ ) and  $\text{A}_2\text{MnSiO}_4$  ( $\text{A} = \text{Na}, \text{Ag}$ ) were compared in Figure 2 with the previously reported structure of



**Figure 2.** Crystal structures of (a, d, and g)  $\text{Li}_2\text{MnSiO}_4$ , (b, e, and h)  $\text{Li}_2\text{MnGeO}_4$ , and (c, f, and i)  $\text{Na}_2\text{MnGeO}_4$ . (a, b, and c) Polyhedral views with the  $\text{AO}_4$ ,  $\text{MnO}_4$ , and  $\text{XO}_4$  tetrahedra in yellow, pink, and gray, respectively. (d, e, and f) Topology of the rigid part of the structures  $(\text{MnXO}_4)^{2-}$ , where Mn and X atoms are pink and gray balls, respectively. The O and A atoms are omitted, the bent  $\text{Mn}–\text{O}–\text{X}$  bridges are shown as black straight lines, and the sphalerite-type pseudocell is outlined in red. (g, h, and i) Packing modes of the magnetic cations Mn are shown as a coordination polyhedron.

$\text{Li}_2\text{MnSiO}_4$  ( $\text{P}2_1/n$ ).<sup>4</sup> The three new  $\text{P}n$  structures are very similar, and  $\text{Na}_2\text{MnGeO}_4$  (Figures 2c, f, and i) effectively represents all of them. Although the three structure types of Figure 2 are essentially different, they have several features in common:

- (1) All of them are based on a double-layered hexagonal close packing of oxygen anions; however, there are no close contacts between anions, so the term “eutaxy”<sup>44</sup> is more appropriate.
- (2) All cations and anions have distorted tetrahedral environments (Figures 2a–c).
- (3) Both  $\text{P}mn2_1$  and  $\text{P}n$  structures may be described as superlattices of the  $\text{ZnO}$  (or wurtzite) type. All cation-centered tetrahedra are in the same orientation, sharing only the vertices. The phases are noncentrosymmetric and hence are piezoelectric. In the  $\text{P}2_1/n$  structure of  $\text{Li}_2\text{MnSiO}_4$ , one-half of the tetrahedra have the opposite orientation to that the other half tetrahedra do, so that

the parameter  $b$  is doubled, the structure is centrosymmetric, and couples of  $\text{LiO}_4$  tetrahedra share edges.

(4) Both monoclinic structures ( $P2_1/n$  and  $Pn$ ) have a three-dimensional (3D) framework, whereas the  $Pmn2_1$  structure is layered (Figures 2d–f).

(5) The  $Pn$  structure type has the topology of a cristobalite where arrangement of Mn and X is of slightly distorted sphalerite type (Figure 2f), so it is natural that the Mn sublattice is a pseudocubic eutaxy in which each Mn has 12 Mn neighbors in the form of a distorted cuboctahedron (Figure 2i). Unexpectedly, essentially the same Mn arrangement is found in the layered  $Pmn2_1$  structure (Figure 2h). In the  $P2_1/n$  structure (Figure 2g), the six nearest Mn neighbors form a distorted trigonal prism with distances to the central Mn of 4.14–4.74 Å and two more neighbors at 5.07 Å capping the prism.

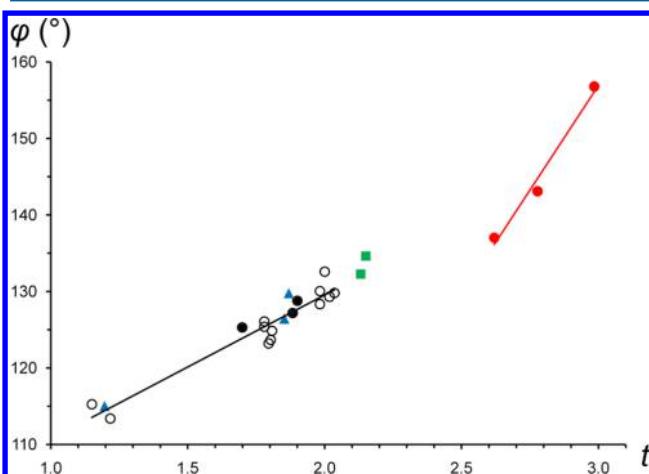
In the four new structures  $\text{Li}_2\text{MnGeO}_4$ ,  $\text{Na}_2\text{MnGeO}_4$ ,  $\text{Na}_2\text{MnSiO}_4$ , and  $\text{Ag}_2\text{MnSiO}_4$ , the longest and shortest Mn–Mn distances within the distorted cuboctahedra (Figures 2h and i) differ by only 20.8, 19.2, 16.8, and 14.3%, respectively (Table 2), demonstrating a gradual approach to the cubic metrics (see below).

The substitution of smaller  $\text{Mn}^{2+}$  for  $\text{Cd}^{2+}$  in the layered structure of  $\text{Li}_2\text{CdGeO}_4$  reduces the lattice parameters  $a$  and  $c$  while either unchanging or slightly increasing the  $b$  parameter (Table S3 of the Supporting Information). This is quite natural because the  $(\text{MGeO}_4)^{2-}$  layers are parallel to the  $ac$  plane, and only the parameters parallel to this layer strongly depend on the size of  $\text{M}^{2+}$ . As the coordination polyhedron of the interlayer ion (here,  $\text{LiO}_4$ ) contracts in the  $ac$  plane, it expands in the perpendicular direction to maintain normal bond lengths; this may compensate for contraction of the rigid layer in the same direction. This behavior is typical of layered structures.<sup>45</sup>

Each oxygen anion in the structures  $\text{A}_2\text{MnXO}_4$  has a distorted tetrahedral environment of two A, one Mn, and one X atom. In the three compounds with large monovalent cations  $\text{Na}^+$  and  $\text{Ag}^+$ , all angles between the strongest bonds, O–X and O–Mn (Table 2), are considerably larger than the ideal tetrahedral value of 109.5°. In  $\text{Li}_2\text{MnGeO}_4$ , this deviation is small, and the same was observed in  $\text{Li}_2\text{MnSiO}_4$  ( $P2_1/n$ ),<sup>4</sup> where the average Si–O–Mn angle is equal to 117.0°. Assuming the lone pair repulsion and/or hard sphere repulsion to be the main factors affecting the bond angles, one would expect exactly the opposite trend. Therefore, alternative explanations are needed. One of them may be purely ionic: the largest angle would be the one between the multiply charged cations. However, a more plausible explanation is based on the structural topology.

In the ZnO-derived structures, the ideal tetrahedral angles require all four cations to be equal in size. With large size differences between mono- and polyvalent cations, the structures are better understood as cristobalite-type  $(\text{MXO}_4)^{2-}$  frameworks stuffed with  $\text{A}^+$ . Increasing the size of the  $\text{A}^+$  cations requires more space for them. The only way to expand the framework with constant X–O and M–O bond lengths is to increase X–O–M angles, approaching the ideal value of 180° found in cubic  $\text{CsAlO}_2$ .<sup>46</sup> This trend was probably first noted in the orthorhombic ( $Pbca$ )  $\text{AFeO}_2$  series ( $\text{A} = \text{K, Rb, Cs}$ )<sup>47</sup> and is further supported by Figure 3 combining the data for all known  $\text{A}_2\text{RXO}_4$  isomorphs of the  $Pn$  type and some

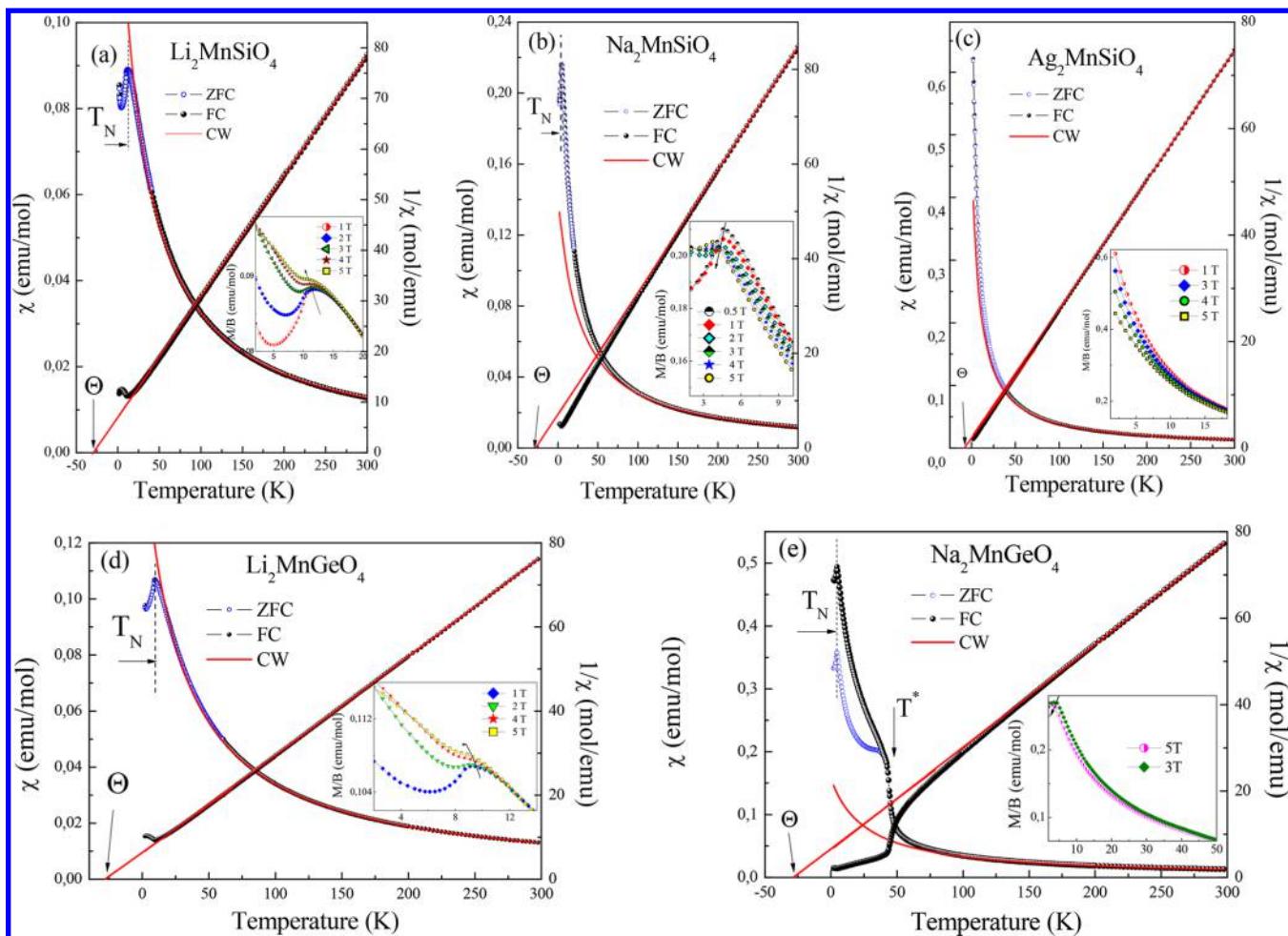
structurally related orthorhombic ( $Pna2_1$ ) gallates and aluminates.



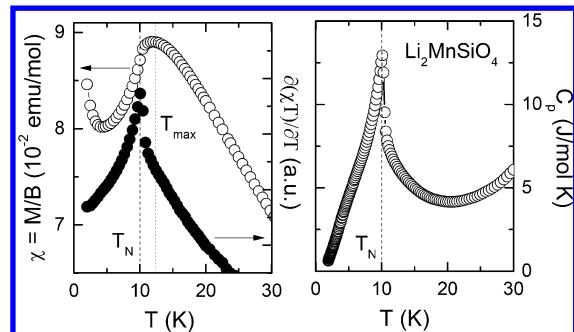
**Figure 3.** Correlation between average bond angle  $\varphi$ , MOX or MOM, and Shannon's<sup>43</sup> radius ratio  $t$  of extra-framework and framework cations in the three families of stuffed cristobalites,  $Pn\text{A}_2\text{MXO}_4$  (black circles),  $Pna2_1\text{AMO}_2$  (blue triangles, M = Ga; green squares, M = Al), and  $Pbca\text{AFeO}_2$  (red circles).  $t = {}^{IV}R(\text{A}^+) \times 2 / [{}^{IV}R(\text{M}^{2+}) + {}^{IV}R(\text{X}^{4+})]$  for  $\text{A}_2\text{MXO}_4$ ,  $t = {}^{IV}R(\text{A}^+) / {}^{IV}R(\text{M}^{3+})$ , and  $t = {}^{VIII}R(\text{A}^+) / {}^{IV}R(\text{Fe}^{3+})$  for  $\text{AFeO}_2$  Phases, left to right:  $\text{Li}_2\text{ZnGeO}_4$ ,<sup>48</sup>  $\text{LiGaO}_2$ ,<sup>49</sup>  $\text{Li}_2\text{MnSiO}_4$ ,<sup>29,39</sup> (by DFT only),<sup>10</sup>  $\text{Na}_2\text{MnGeO}_4$ ,<sup>24</sup>  $\text{Na}_2\text{ZnGeO}_4$ ,<sup>50</sup> (two points),  $\text{Ag}_2\text{ZnGeO}_4$ ,<sup>24</sup>  $\text{Li}_2\text{BeSiO}_4$ ,<sup>51</sup>  $\text{Na}_2\text{CoGeO}_4$ ,<sup>52</sup>  $\text{NaGaO}_2$ ,<sup>53</sup>  $\text{AgGaO}_2$ ,<sup>53</sup>  $\text{Na}_2\text{MnSiO}_4$ ,<sup>14b</sup>  $\text{Ag}_2\text{MnSiO}_4$ ,<sup>19</sup>  $\text{Na}_2\text{ZnSiO}_4$ ,<sup>39,40</sup>  $\text{Ag}_2\text{ZnSiO}_4$ ,<sup>24</sup>  $\text{Na}_2\text{CoSiO}_4$ ,<sup>47</sup>  $\text{Na}_2\text{MgSiO}_4$ ,<sup>54</sup>  $\text{NaAlO}_2$ ,<sup>55</sup>  $\text{AgAlO}_2$ ,<sup>55</sup>  $\text{KFeO}_2$ ,<sup>27</sup>  $\text{RbFeO}_2$ ,<sup>47</sup>  $\text{CsFeO}_2$ .<sup>47</sup> Filled black circles and formulas in bold correspond to the three new phases from the present work. The data for  $\text{Li}_2\text{CoGeO}_4$  ( $Pn$ )<sup>18</sup> were not included because of impossible interatomic distances.

Interestingly enough, the trend toward the ideal cubic structure, evident from the evolution of X–O–Mn bond angles and Mn–Mn distance ratios discussed above, could not be deduced from the evolution of the lattice parameters. The ratios of the three pseudocubic cell edges do not show regular approaching unity on silver substitution for sodium in  $\text{Na}_2\text{RSiO}_4$  (R = Zn, Mn) and  $\text{Na}_2\text{ZnGeO}_4$  (see Table S3 of the Supporting Information) as well as in the  $\text{AFeO}_2$  series (A = K, Rb, Cs).<sup>47</sup> On heating, too, the pseudocubic cell edges of  $\text{KFeO}_2$  and  $\text{RbFeO}_2$  initially diverge and converge only in the vicinity of the orthorhombic-to-cubic phase transition.<sup>47</sup>

**3.3. Magnetic Susceptibility.** The temperature-dependent magnetic susceptibilities  $\chi = M/B$  determined for  $\text{Li}_2\text{MnSiO}_4$ ,  $\text{Na}_2\text{MnSiO}_4$ ,  $\text{Ag}_2\text{MnSiO}_4$ ,  $\text{Li}_2\text{MnGeO}_4$ , and  $\text{Na}_2\text{MnGeO}_4$  are presented in Figure 4. Except for  $\text{Ag}_2\text{MnSiO}_4$ , the  $\chi$  vs  $T$  plots exhibit a sharp peak at a low temperature, suggesting the occurrence of a long-range antiferromagnetic (AFM) ordering. A closer look at the data displays a broad susceptibility maximum  $\chi_{\max}$  at  $T_{\max}$  and a sharp peak at slightly lower temperature. The magnetic specific heat,  $\partial(\chi T)/\partial T$ , shows a pronounced  $\lambda$ -shape anomaly (for example, see Figure 5 for  $\text{Li}_2\text{MnSiO}_4$ ). This allows one to determine the Néel temperature  $T_N$ , below which a long-range AFM ordering takes place. Our analysis shows that  $T_N = 10.3 \pm 0.5$  for  $\text{Li}_2\text{MnSiO}_4$ ,  $4.1 \pm 0.5$  K for  $\text{Na}_2\text{MnSiO}_4$ ,  $8.1 \pm 0.5$  K for  $\text{Li}_2\text{MnGeO}_4$ , and  $4.0 \pm 0.5$  K for  $\text{Na}_2\text{MnGeO}_4$ . The  $T_{\max}$  appears at 0.5 to 2 K above  $T_N$ . Such a behavior is found for low-dimensional magnetic systems for which the  $\chi_{\max}$  arises from a short-range AFM



**Figure 4.** Temperature dependence of the magnetic susceptibility and its inverse measured at  $B = 0.1$  T for (a)  $\text{Li}_2\text{MnSiO}_4$ , (b)  $\text{Na}_2\text{MnSiO}_4$ , (c)  $\text{Ag}_2\text{MnSiO}_4$ , (d)  $\text{Li}_2\text{MnGeO}_4$ , and (e)  $\text{Na}_2\text{MnGeO}_4$ . The red solid curves represent an approximation in accordance with the Curie–Weiss law.



**Figure 5.** (a) Magnetic susceptibility obtained at  $B = 0.1$  T, associated magnetic specific heat, and (b) specific heat of  $\text{Li}_2\text{MnSiO}_4$  at  $T \leq 30$  K. The dashed line shows the position of the  $\lambda$ -like anomaly at  $T_N$ ; the dotted line indicates a broad correlation maximum in  $\chi$  at  $T_{\text{max}}$  (see the text).

ordering that takes place above  $T_N$  and the long-range order occurs at temperatures lower than those suggested by mean field theory.<sup>56</sup> In contrast, the magnetic susceptibility of  $\text{Ag}_2\text{MnSiO}_4$  shows a Curie–Weiss behavior over the whole temperature range down to 1.8 K, implying the absence of a long-range magnetic order (Figure 4c).

The temperature-dependent magnetic susceptibility of  $\text{Na}_2\text{MnGeO}_4$  shows a rather complicated behavior with additional anomaly at  $T^* \approx 43$  K (Figure 4e), which is

associated with an increase in the magnetization by  $\sim 3 \times 10^{-3} \mu_B$  per formula unit (f.u.). There occurs a clear discrepancy between the ZFC and FC curves at  $T < T^*$ . Correspondingly, the inverse susceptibility deviates strongly from the Curie–Weiss behavior on approaching the  $T^*$  anomaly. In applied magnetic fields, the  $T_N$  shifts slightly to lower temperatures for all samples, and the  $T^*$  anomaly disappears (see the inset of Figure 4e). Therefore, this anomaly can be related to the presence of a ferromagnetic impurity, most probably a small amount of  $\text{Mn}_3\text{O}_4$ . Indeed, a careful inspection of the XRD pattern revealed the presence of very weak reflections from  $\text{Mn}_3\text{O}_4$  (and also from  $\text{MnO}$ ) not detected in our initial XRD study and not accounted for in the Rietveld analysis. Exactly the same situation was described in a recent investigation of  $\text{Mn}_2\text{Sb}_2\text{O}_7$ : a minor impurity of  $\text{Mn}_3\text{O}_4$  was overlooked by XRD but detected by magnetic measurements.<sup>57</sup>

For all five phases  $\text{Li}_2\text{MnSiO}_4$ ,  $\text{Na}_2\text{MnSiO}_4$ ,  $\text{Ag}_2\text{MnSiO}_4$ ,  $\text{Li}_2\text{MnGeO}_4$ , and  $\text{Na}_2\text{MnGeO}_4$ , the magnetic susceptibilities at  $T > 70$  K are nicely described by the Curie–Weiss law:

$$\chi = \chi_0 + \frac{C}{T - \Theta} \quad (1)$$

where  $\chi_0$  is the temperature-independent susceptibility,  $\Theta$  the Weiss temperature, and  $C$  the Curie constant  $C = N_A \mu_{\text{eff}}^2 \mu_B^2 / 3k_B$  ( $N_A$  = Avogadro's number,  $\mu_{\text{eff}}$  = the effective magnetic moment,  $\mu_B$  = Bohr magneton, and  $k_B$  = Boltzmann constant).

**Table 3.** Values of  $T_N$  (in K),  $\Theta$  (in K), and  $\mu_{\text{eff}}$  (in  $\mu_B$ ) of  $A_2\text{MnXO}_4$  from Magnetic Susceptibility Measurements and  $g$ -Factor of  $A_2\text{MnXO}_4$  from ESR Measurements

	$T_N$	$\Theta$	$f =  \Theta /T_N$	$\mu_{\text{eff}}$	$g$
$\text{Li}_2\text{MnSiO}_4$	10.1	$-30 \pm 1$	3.0	5.78	1.99
	12 <sup>33</sup>	-38	3.2	5.7	
	12 <sup>6</sup>	-29	2.4	6	2.003
	12 <sup>32</sup>	-35	2.9	6	2.00 <sup>11</sup>
	12 <sup>7</sup>	-29.4	2.5	5.85	
$\text{Na}_2\text{MnSiO}_4$	$4.2 \pm 0.5$	$-27 \pm 1$	6.4	5.58	1.99
$\text{Ag}_2\text{MnSiO}_4$		$-9 \pm 1$	>5	5.78	2.01
$\text{Li}_2\text{MnGeO}_4$	8.1	$-27 \pm 1$	3.5	5.89	1.99
$\text{Na}_2\text{MnGeO}_4$	4.0	$-27 \pm 1$	6.8	5.81	1.99

We estimated  $\chi_0$  by summing Pascal's constants.<sup>58</sup> The parameters for the five phases extracted from fitting the experimental data in the range 200–300 K are summarized in Table 3, and those of the related silicates and germanates available in the literature are in Table S4 of the Supporting Information. The effective magnetic moments  $\mu_{\text{eff}}$  calculated from the corresponding Curie constants were found to be in reasonable agreement with theoretical estimates  $\mu_{\text{theor}}^2 = g^2\mu_B^2S(S+1)$  using the effective  $g$ -factors (Table 3) determined from the ESR data (see below). Our analysis yields negative  $\Theta$  values for all five compounds, indicating the presence of predominant AFM interactions. For classical 3D AFM systems, frustration index  $f = |\Theta|/T_N$  usually takes values of 1–3. Considerably larger values would signal a presence of the marked spin frustration. Obviously, such frustration can hamper the onset of the long-range ordering.<sup>59</sup> The  $f$  values of  $\text{Li}_2\text{MnSiO}_4$  and  $\text{Li}_2\text{MnGeO}_4$  are smaller than 4, so it is not surprising that they undergo a 3D AFM ordering.  $\text{Na}_2\text{MnSiO}_4$  and  $\text{Na}_2\text{MnGeO}_4$ , however, have much larger  $f$  values (6.4 and 6.8, respectively), yet they also undergo a 3D AFM ordering. The  $T_N$  values of  $\text{Na}_2\text{MnSiO}_4$  and  $\text{Na}_2\text{MnGeO}_4$  are  $\sim 4$  K, which are approximately half the corresponding values of their Li-analogues (10.1 and 8.1 K, respectively) (Table 3).

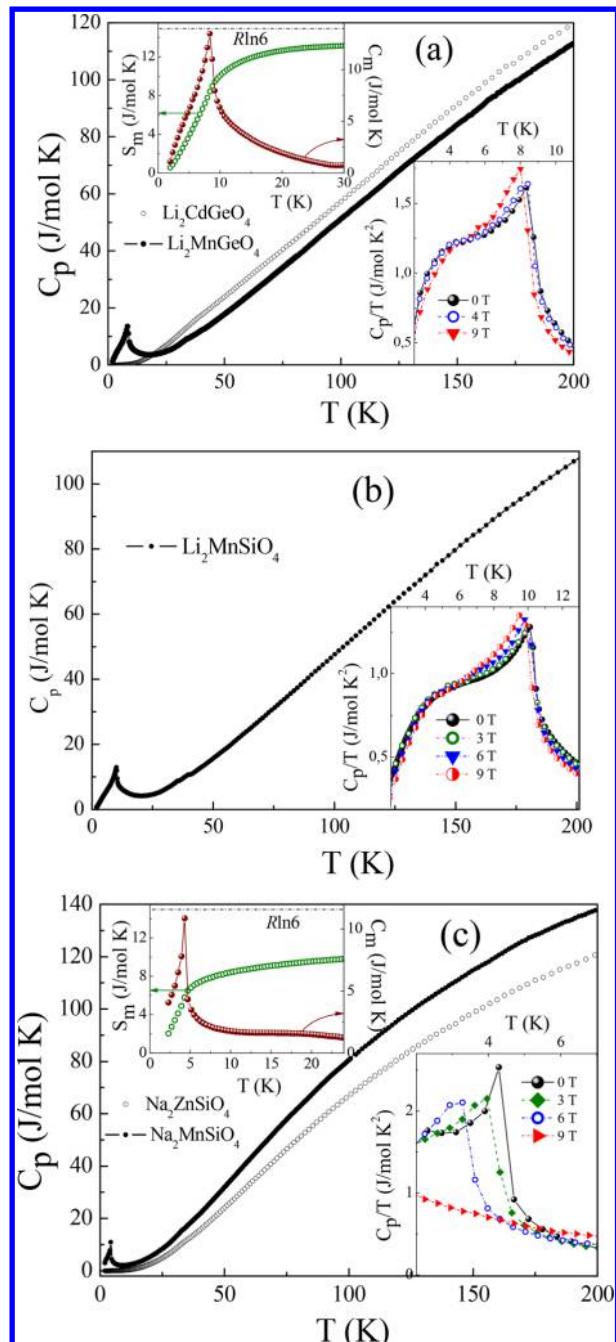
The possible presence of ferromagnetic (FM) correlations in  $\text{Na}_2\text{MnSiO}_4$  and  $\text{Na}_2\text{MnGeO}_4$  can be inferred from the way their experimental  $\chi(T)$  curves deviate from the Curie–Weiss law at low temperatures (Figures 4b, c, and e). A ferromagnetic (FM) contribution to the magnetization was pointed out earlier for other related compounds, particularly for  $\text{Li}_2\text{FeSiO}_4$ <sup>3,32</sup> and  $\text{Li}_2\text{CoSiO}_4$ ,<sup>33</sup> and has been related to the presence of FM impurity phases by XRD data. For  $\text{Li}_2\text{MnSiO}_4$ ,  $\text{Na}_2\text{MnSiO}_4$ ,  $\text{Ag}_2\text{MnSiO}_4$ , and  $\text{Li}_2\text{MnGeO}_4$ , however, our magnetic studies (e.g., a visible hysteresis between ZFC and FC  $\chi(T)$  curves) do not show any direct indication of such impurity. The available theoretical calculations also support such a scenario because they yield a very low energy difference ( $\sim 1$  meV)<sup>60</sup> between FM and AFM quantum ground states for the Li–M<sup>2+</sup> silicates and predict very similar effective magnetic moment values per high-spin transition-metal ions regardless of either the FM or AFM approach.<sup>60–64</sup> Moreover, as it was experimentally shown by Avdeev et al. from the neutron scattering studies, a quite complicated spin configuration is realized in magnetic sublattice for both  $\text{Li}_2\text{MnSiO}_4$  and  $\text{Li}_2\text{CoSiO}_4$ , which contain AFM zigzag chains running along [001] and [101], respectively, forming corrugated FM layers of hexagons stacked perpendicular to these directions.<sup>33</sup>

The  $T_N$ ,  $\Theta$ , and  $\mu_{\text{eff}}$  values obtained for  $\text{Li}_2\text{MnSiO}_4$  in the present work are in reasonable agreement with those reported

in the literature regardless of its polymorph structure (Table 3). This corroborates the conclusion of Bini et al. that the  $T_N$  value is influenced by the coexistence between transition ions in different oxidation states and the presence of the impurities rather than the polymorphism of Li–M<sup>2+</sup> silicates.<sup>32</sup> Some difference in the  $\Theta$  values (−29.4 to −38 K) is presumably caused by the uncertainty in the estimation of  $\chi_0$  and the different temperature regimes used for the Curie–Weiss fitting. The largest discrepancy in the reported  $\Theta$  values was found for  $\text{Li}_2\text{FeSiO}_4$ , namely,  $\Theta = -35$  to  $-93.5$  K<sup>3,7,32</sup> (Table S4 of the Supporting Information).

**3.4. Specific Heat.** The specific heat was measured for  $\text{Li}_2\text{MnGeO}_4$ ,  $\text{Na}_2\text{MnSiO}_4$ , and  $\text{Li}_2\text{MnSiO}_4$ , which undergo a 3D AFM ordering as well as for nonmagnetic structural analogues of the first couple of phases:  $\text{Li}_2\text{CdGeO}_4$  and  $\text{Na}_2\text{ZnSiO}_4$ . The specific heat data  $C(T)$  for all Mn compounds at zero magnetic field exhibit a distinct  $\lambda$ -anomaly, which is a characteristic feature for a 3D AFM ordering (Figure 6). The values of  $T_N$ , deduced from  $C(T)$  data at  $B = 0$  T, are listed in Table 4. In applied magnetic fields, the  $\lambda$ -anomaly is slightly rounded and shifts to the lower temperature (see the lower right insets in Figure 6). To analyze the magnetic phase transition and evaluate the corresponding contribution to the specific heat and entropy in  $\text{Li}_2\text{MnGeO}_4$  and  $\text{Na}_2\text{MnSiO}_4$ , we evaluated the lattice contribution to the specific heat by using the isostructural diamagnetic analogues (i.e.,  $\text{Li}_2\text{CdGeO}_4$  and  $\text{Na}_2\text{ZnSiO}_4$ , respectively). The Dulong–Petit value is  $3R\nu = 199.54$  J/(mol K), where  $R$  is the gas constant  $R = 8.314$  J/(mol K) and  $\nu$  is the number of atoms per formula unit (in the present case,  $\nu = 8$ ). As reported in Table 4, the magnitude of the  $\lambda$ -anomaly observed at  $T_N$  is noticeably smaller (1.5–1.8 times) than the one expected from the mean-field theory for the 3D antiferromagnetic ordering of a magnetic system with high-spin Mn<sup>2+</sup> ion ( $S = 5/2$ ),<sup>65</sup> i.e.,  $\Delta C_p = 5R\nu S(S+1)/[(S+1)^2 + S^2] = 19.66$  J/(mol K). Such a reduction of  $\Delta C_p$  suggests that  $\text{Li}_2\text{MnGeO}_4$ ,  $\text{Na}_2\text{MnSiO}_4$ , and  $\text{Li}_2\text{MnSiO}_4$  are low-dimensional magnets and that a short-range AFM ordering is present above  $T_N$ .

Assuming that the  $C(T)$  of a diamagnetic material provides a proper estimation for the pure lattice contribution to the specific heat, the magnetic contribution  $C_m(T)$  can be estimated by using the scaling procedure.<sup>66</sup> In Figure 6a (6c) we compare the temperature dependence of the total specific heat for magnetic  $\text{Li}_2\text{MnGeO}_4$  ( $\text{Na}_2\text{MnSiO}_4$ ) and that of diamagnetic  $\text{Li}_2\text{CdGeO}_4$  ( $\text{Na}_2\text{ZnSiO}_4$ ) in the 160–280 K range. From these, we calculate the lattice part  $C_{\text{ph}}(T)$  for the magnetic phase in the range 2–70 K. The Debye temperatures  $\Theta_D$ , estimated using the  $C(T) \propto T^3$  law in low temperature



**Figure 6.** Temperature dependence of the specific heat  $C_p$  of (a)  $\text{Li}_2\text{MnGeO}_4$  and its structural analogue  $\text{Li}_2\text{CdGeO}_4$ , (b)  $\text{Li}_2\text{MnSiO}_4$ , and (c)  $\text{Na}_2\text{MnSiO}_4$  and its structural analogue  $\text{Na}_2\text{ZnSiO}_4$  in zero magnetic field. Insets: (lower right) temperature dependence of  $C_p/T$  under different magnetic fields and (upper left) temperature dependence of  $C_m(T)$  and the entropy  $S_m(T)$ .

range (2–18 K), are summarized in **Table 4**. The Debye temperatures for the magnetic systems were estimated by using the normalization procedure<sup>65</sup> in which the difference between the molar masses for Zn–Mn or Cd–Mn atoms is taken into account.

The magnetic entropy change was calculated using the equation:

$$\Delta S_m(T) = \int_0^T \frac{C_m(T)}{T} dT \quad (2)$$

**Table 4. Thermodynamic Parameters of  $\text{A}_2\text{MnXO}_4$  from Specific Heat Measurements**

compound	$T_N$ (K)	$\theta_D$ (K)	$\Delta S$ (J/mol K)	$\Delta C$ (J/mol K)	molar mass (g/mol)
$\text{Li}_2\text{MnGeO}_4$	$8.1 \pm 0.5$	403.5	13.3	13.44	205.46
$\text{Li}_2\text{CdGeO}_4$		352.5			262.93
$\text{Na}_2\text{MnSiO}_4$	$4.2 \pm 0.5$	394	10.6	10.88	193.00
$\text{Na}_2\text{ZnSiO}_4$		388			203.45
$\text{Li}_2\text{MnSiO}_4$	$10.1 \pm 0.5$			12.5	160.90

which converges to a value smaller in magnitude than the one expected from the mean-field theory,  $\Delta S_m = 2R\ln(2S + 1) \approx 14.9 \text{ J}/(\text{mol K})$  (the dashed line in the upper left insets in **Figure 6**). The magnetic entropy released below  $T_N$  removes only about 30–40% of the saturation value, which is characteristic feature for a magnetic system with lower magnetic dimensionality.<sup>67</sup>

**3.5. Field Dependence of the Magnetization.** The magnetization curves  $M(B)$  vs  $B$  for  $\text{Li}_2\text{MnGeO}_4$ ,  $\text{Li}_2\text{MnSiO}_4$ ,  $\text{Na}_2\text{MnSiO}_4$ , and  $\text{Na}_2\text{MnGeO}_4$  determined at various temperatures in magnetic fields up to 5 T are presented in **Figure 7**. In this range of applied fields, the maximum of the magnetic moment remains essentially lower than the theoretically expected saturation magnetic moment for the high-spin  $\text{Mn}^{2+}$  ions ( $S = \frac{5}{2}$ ):  $M_s = gS\mu_B \approx 5 \mu_B/\text{f.u.}$  The magnetization curves exhibit a clear upward curvature, suggesting the presence of a field-induced spin-reorientation (spin-flop type) transition. The critical fields,  $B_{SF}$ , were determined from the maximum of the first derivative,  $dM(B)/dB$ , as indicated by arrows in **Figure 7**, and were used in constructing the magnetic phase diagrams (see **Section 3.7**). At  $T = 2 \text{ K}$ ,  $B_{SF} \approx 1.6 \text{ T}$  for  $\text{Li}_2\text{MnGeO}_4$ , 2.2 T for  $\text{Li}_2\text{MnSiO}_4$ , and 1.8 T for  $\text{Na}_2\text{MnSiO}_4$ . Once again, the dependence  $M(B)$  for  $\text{Na}_2\text{MnGeO}_4$  demonstrates a more complicated character and reveals the presence of at least two different features at  $B_{SF} \approx 1.6 \text{ T}$  and  $B_{C1} \approx 0.8 \text{ T}$  at 2 K (**Figure 7d**). While  $B_{SF}$  vanishes above  $T_N$ , the second anomaly  $B_{C1}$  survives up to  $T^* \approx 42 \text{ K}$ . Hence, it is reasonable to relate it to the FM impurity  $\text{Mn}_3\text{O}_4$ . With increasing temperature, the  $B_{SF}$  anomaly slightly shifts to higher fields, weakens in amplitude, and eventually disappears above  $T_N$  for all compounds.

The magnetization,  $M$  vs  $B$ , for the  $\text{Ag}_2\text{MnSiO}_4$  sample (**Figure 8**) shows a rather different behavior. There is a slight bending in the  $B$  vs  $T$  curve, which is weaker than expected for a  $S = \frac{5}{2}$  paramagnet. The magnetization tends to saturate at small magnetic fields reaching the maximum value of  $4.5 \mu_B/\text{Mn}^{2+}$  at 5 T, which is close to the theoretically expected full saturation value of the magnetization. The magnetization is well-described by a Brillouin function if a significant AFM mean field is considered (see **Figure 8**), which is phenomenologically given by

$$M = N_{\text{Ag}}\mu_B B S B_S \left( \frac{g\mu_B B S}{k_B(T - \Theta^*)} \right) + \chi_0 T \quad (3)$$

where the parameter  $\Theta^*$  is related to short-range AFM correlations between Mn spins in  $\text{Ag}_2\text{MnSiO}_4$ . The least-squares fitting according to **eq 3** yields the best agreement with experimental data by using  $g = 2.01$ ,  $S = \frac{5}{2}$ , and  $\Theta^* \approx -5 \text{ K}$  (with  $\chi_0 = -12 \times 10^{-5} \text{ emu/mol}$  used for **eq 1**). The data hence confirm the absence of a long-range AFM order but imply the presence of AFM correlations at  $T = 2 \text{ K}$ .  $\Theta^* \approx -5 \text{ K}$

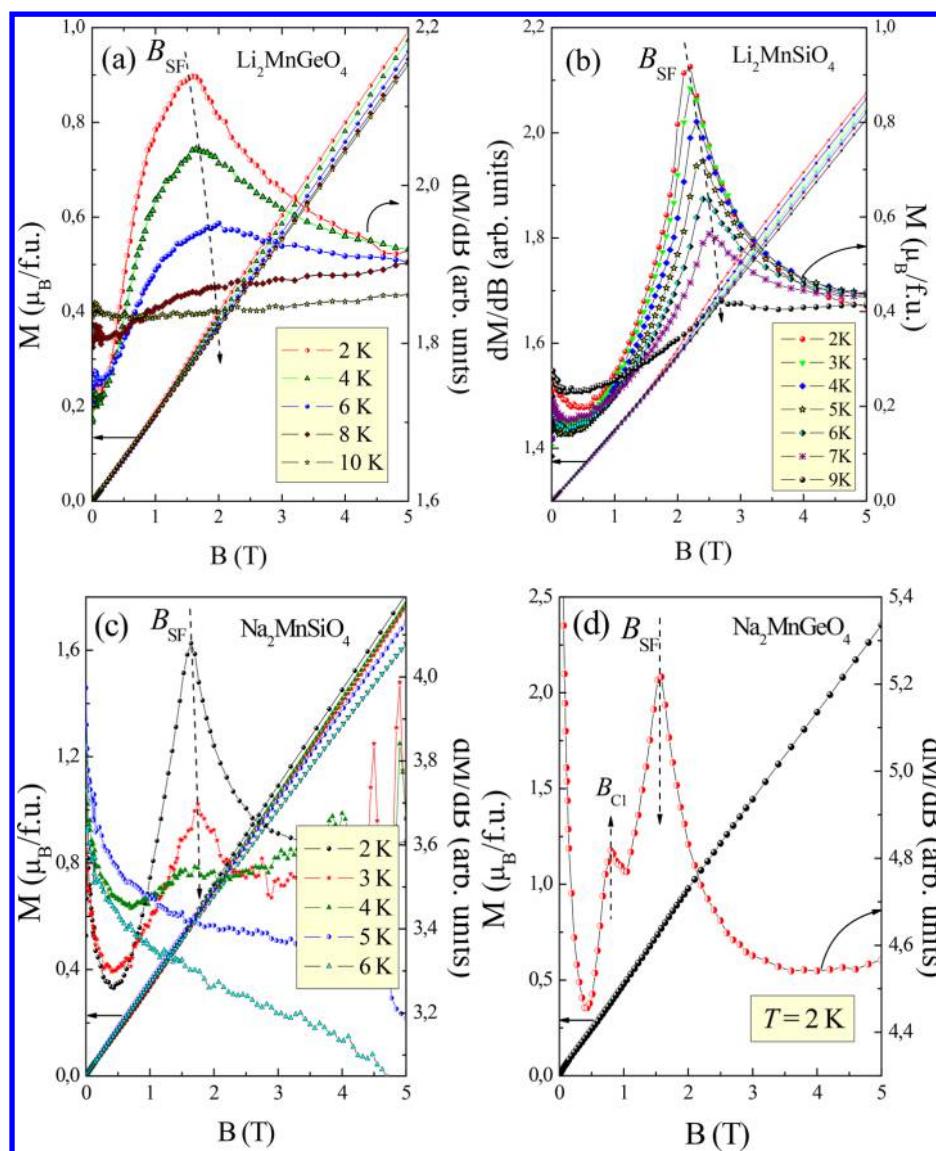


Figure 7.  $M$  vs  $B$  and  $dM$  vs  $dB$  plots determined for (a)  $\text{Li}_2\text{MnGeO}_4$ , (b)  $\text{Li}_2\text{MnSiO}_4$ , (c)  $\text{Na}_2\text{MnSiO}_4$ , and (d)  $\text{Na}_2\text{MnGeO}_4$  at several different temperatures. The arrows point the positions of the field-induced phase transitions.

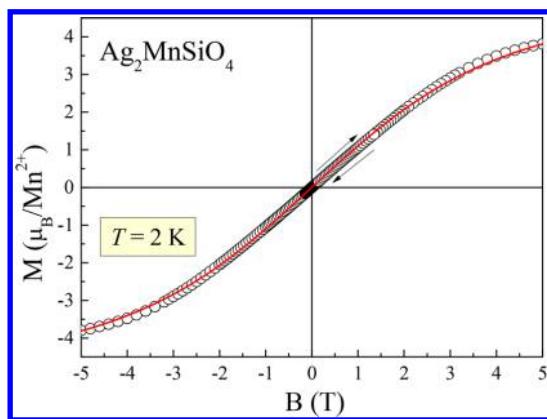
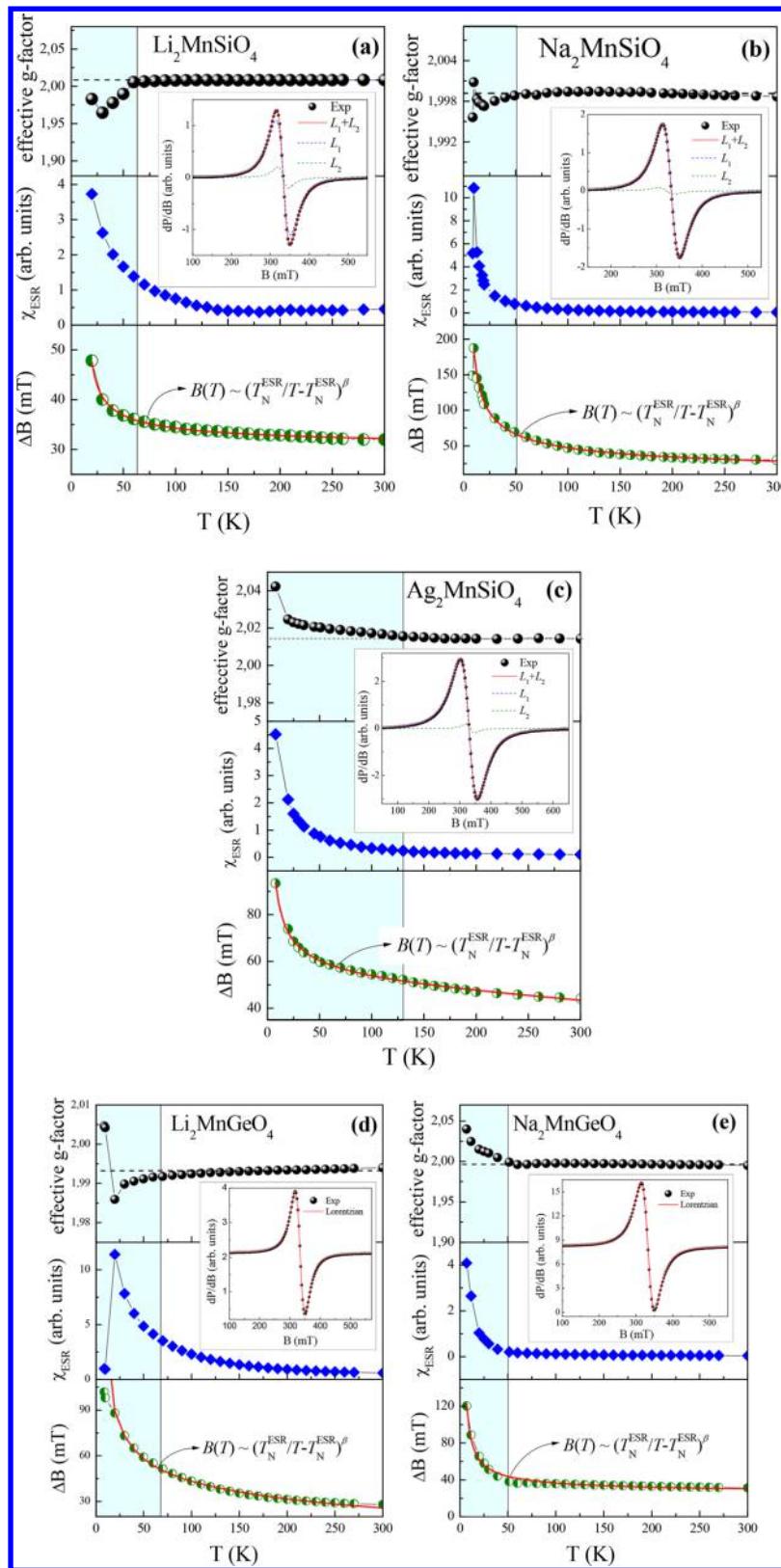


Figure 8.  $M$  vs  $B$  plot measured for  $\text{Ag}_2\text{MnSiO}_4$  at  $T = 2$  K. The circles are experimental data, and the red solid curve is the result of eq 3 (see text).

is consistent with the Curie–Weiss temperature  $\Theta = -9$  K (Table 3).

**3.6. Electron Spin Resonance.** All  $\text{A}_2\text{MnXO}_4$  ( $\text{A} = \text{Li}, \text{Na}, \text{Ag}; \text{X} = \text{Si}, \text{Ge}$ ) samples under study show a similar ESR response in the paramagnetic phase ( $T > T_N$ ) and reveal a single exchange-narrowed absorption line ascribable to  $\text{Mn}^{2+}$  ions in tetrahedral coordination (Figure 9). A thorough analysis shows that for the better description of the line shape in the case of the  $\text{A}_2\text{MnSiO}_4$  silicates, it is necessary to use two Lorentzians with significantly different relative intensities (for representative examples, see the insets of Figures 9a–c). The mode  $L_1$  shown in blue in the insets of Figure 9a–c represents the main contribution to the absorption, and the second small absorption mode can be ascribed to the presence of a small amount of impurities in the samples. The main ESR parameters (namely, the effective  $g$ -factor, the ESR line width, and the integral ESR intensity) were deduced by fitting the experimental spectra with either one (for  $\text{A}_2\text{MnGeO}_4$ ) or two (for  $\text{A}_2\text{MnSiO}_4$ ) Lorentzian profiles to take into consideration the two circular components of the exciting linearly polarized microwave field on both sides of  $B = 0$  because the ESR line is relatively broad,



**Figure 9.** Temperature dependence of the effective *g*-factor (upper panels), the integral ESR intensity (middle panels), and the ESR line width (lower panels) of  $A_2MnXO_4$ . The red solid curves are the prediction from the Mori–Kawasaki–Huber theory (see the text). Insets: Representative examples of ESR spectra fitting with two Lorentzians at (a)  $T = 250$ , (b)  $230$ , and (c)  $200$  K, and with one Lorentzian at (d and e)  $T = 250$  K. The black circles are experimental data; the blue and green curves are resolved components, and the red solid curve is their sum. The light blue regions show the temperature range where the presence of short-range correlations is obvious from the deviation of the *g*-value.

$$\frac{dP}{dB} \propto \frac{d}{dB} \left[ \frac{\Delta B}{\Delta B^2 + (B - B_r)^2} + \frac{\Delta B}{\Delta B^2 + (B + B_r)^2} \right] \quad (4)$$

where  $P$  is the power absorbed in the ESR experiment,  $B$  the magnetic field,  $B_r$  the resonance field, and  $\Delta B$  the line width.

The parameters of the main resonance mode  $L_1$  deduced from the fitting are collected in Figure 9 for all samples. The overall temperature behavior of the ESR parameters corroborates well with that of the static magnetic properties for all samples. The integral ESR intensity  $\chi_{\text{esr}}$ , which is proportional to the number of magnetic spins, was estimated by double integration of the first derivative ESR spectrum  $dP/dB$ . The  $\chi_{\text{esr}}$  follows the Curie–Weiss relationship and agrees with the behavior of the magnetic susceptibility  $\chi$ . The parameters for  $\text{A}_2\text{MnXO}_4$  ( $\text{A} = \text{Li}, \text{Na}$ ) that undergo a long-range AFM order above 2 K exhibit anomalies in the vicinity of  $T_N$ . With decreasing temperature below  $\sim 100$  K, the absorption line broadens markedly and passes through a maximum at about  $T_N$ . The effective  $g$ -factor remains almost temperature-independent down to 50–70 K for different samples ( $g \approx 2$  for all samples) (see also Table 3). When the temperature decreases further, the  $g$ -factor deviates markedly from its high-temperature value. This indicates an increasing role of the short-range correlations upon approaching an order–disorder transition from above. It should be stressed that the value  $g = 2.003$  at  $T = 5$  K reported for  $\text{Li}_2\text{MnSiO}_4$  ( $Pmn_2$ ) by Belharouak et al.<sup>6</sup> from X-band ESR measurements does not reflect the paramagnetic (spectroscopic)  $g$ -factor because it was determined well below  $T_N = 12$  K (i.e., in the long-range AFM-ordered state), where the resonance reflects collective excitations or, if probing the impurity spins, are affected by the internal exchange and anisotropy fields of the AFM phase.

The effective  $g$ -factor and the ESR line width demonstrate a more complicated behavior for  $\text{Na}_2\text{MnGeO}_4$ , and an additional anomaly around  $T^*$  can be seen. Below  $T^*$ , the description for the line shape of the experimental ESR spectra requires the use of two Lorentzians. Thus, similarly to thermodynamic data, the ESR data indicate the presence of impurity, most probably  $\text{Mn}_3\text{O}_4$ . The ESR parameters for  $\text{Ag}_2\text{MnSiO}_4$  are similar to those of other  $\text{A}_2\text{MnXO}_4$  ( $\text{A} = \text{Li}, \text{Na}; \text{X} = \text{Si}, \text{Ge}$ ) but show greater short-range correlation effects in an essentially wider temperature range (i.e., at  $T < 130$  K) and do not show any nonmonotonous behavior. This is consistent with the absence of a long-range AFM order within the temperature range studied, and the finding for short-range correlations confirms the result of the magnetization analysis.

The broadening of the ESR line upon approaching  $T_N$  may be treated in terms of a critical behavior of the ESR line width, which is caused by the slowing of spin fluctuations in the vicinity of an order–disorder transition.<sup>68–71</sup> This causes the divergence of the spin correlation length, which in turn affects the spin–spin relaxation time of exchange-narrowed ESR lines resulting in the critical broadening:

$$\Delta B(T) = \Delta B^* + A \left[ \frac{T_N^{\text{ESR}}}{T - T_N^{\text{ESR}}} \right]^\beta \quad (5)$$

where the first term  $\Delta B^*$  describes the exchange-narrowed line width, which is temperature-independent. The second term describes the critical behavior with  $T_N^{\text{ESR}}$  as the temperature of the order–disorder transition and  $\beta$  as the critical exponent. To account for the  $\Delta B$  behavior of  $\text{Ag}_2\text{MnSiO}_4$  over the high

temperature range, it is necessary to include an additional third linear term into this fitting formula. The solid red curves on the lower panels of Figure 9 represent a least-squares fitting of the  $\Delta B(T)$  experimental data using eq 5. The best fitting was obtained with the parameters listed in Table 5. The value of

**Table 5. Fitting Parameters for the Temperature Dependence of the ESR Line Width  $\Delta B$  for  $\text{A}_2\text{MnXO}_4$  in Accordance with Eq 5**

	$T_N^{\text{ESR}}$ (K)	$\Delta B^*$ (mT)	$\beta$
$\text{Li}_2\text{MnSiO}_4$	$13.3 \pm 0.5$	$30 \pm 1$	$0.55 \pm 0.05$
$\text{Na}_2\text{MnSiO}_4$	$4.6 \pm 0.5$	$4 \pm 1$	$0.50 \pm 0.05$
$\text{Ag}_2\text{MnSiO}_4$	$0.01 \pm 0.01$	$48 \pm 1$	$0.66 \pm 0.05$
$\text{Li}_2\text{MnGeO}_4$	$6.5 \pm 0.5$	$2 \pm 1$	$0.37 \pm 0.05$
$\text{Na}_2\text{MnGeO}_4$	$3.1 \pm 0.5$	$24 \pm 1$	$0.65 \pm 0.05$

$T_N^{\text{ESR}}$  is in a reasonable agreement with the  $T_N$  obtained from the static magnetic susceptibility measurements. The critical exponents  $\beta$  for  $\text{A}_2\text{MnSiO}_4$  ( $\text{A} = \text{Li}, \text{Na}, \text{Ag}$ ) and  $\text{Na}_2\text{MnGeO}_4$  vary in the range of 0.55–0.66 except for  $\text{Li}_2\text{MnGeO}_4$ , where  $\beta = 0.37 \pm 0.05$  (Table 5). The theoretical  $\beta$  value expected for 3D Heisenberg antiferromagnets is  $1/3$ . This indicates either a low-dimensional or frustrated character of magnetism for  $\text{A}_2\text{MnSiO}_4$  ( $\text{A} = \text{Li}, \text{Na}, \text{Ag}$ ) and  $\text{Na}_2\text{MnGeO}_4$ .

**3.7. Magnetic Phase Diagrams.** By summarizing the anomalies observed from the field and temperature dependencies of the magnetization in the present work, we constructed the magnetic phase diagrams for  $\text{A}_2\text{MnXO}_4$  (Figure 10). The phases  $\text{A}_2\text{MnGeO}_4$  and  $\text{A}_2\text{MnSiO}_4$  ( $\text{A} = \text{Li}, \text{Na}$ ) have a long-range AFM ordered ground state. The weak magnetic anisotropy of the  $\text{Mn}^{2+}$  ( $S = 5/2$ ) ions leads to spin-flop transitions at rather low magnetic fields, i.e., at around 2 T in compounds studied. Accordingly, the associated anisotropy gap is of the order of  $\Delta \approx g\mu_B B_{\text{SF}} (T = 0) \approx 0.23$  meV. An additional anomaly  $B_{\text{C}1}$  in  $\text{Na}_2\text{MnGeO}_4$  is related most probably to the presence of a small  $\text{Mn}_3\text{O}_4$  impurity in the sample, as already mentioned.

**3.8. Analyses of the Magnetic Properties in Terms of Spin Exchanges.** In this section, we discuss the magnetic properties of  $\text{A}_2\text{MnSiO}_4$  ( $\text{A} = \text{Li}, \text{Na}, \text{Ag}$ ) and  $\text{A}_2\text{MnGeO}_4$  ( $\text{A} = \text{Li}, \text{Na}$ ) on the basis of their spin exchange interactions, which we evaluate by performing the energy-mapping analysis based on DFT calculations.<sup>36,37</sup>

**3.8.1. Computational Details.** We carried out spin-polarized DFT calculations using a (2a, b, 2c) supercell for  $\text{Li}_2\text{MnSiO}_4$  and a (2a, 2b, 2c) supercell for  $\text{Na}_2\text{MnSiO}_4$ ,  $\text{AgMnSiO}_4$ ,  $\text{Li}_2\text{MnGeO}_4$ , and  $\text{Na}_2\text{MnGeO}_4$  by employing the projected augmented wave method encoded in the Vienna Ab Initio Simulation Package (VASP)<sup>72–74</sup> with the generalized gradient approximation of Perdew, Burke, and Ernzerhof<sup>75</sup> for the exchange-correlation functionals with a plane wave cutoff energy of 520 eV, a set of  $2 \times 2 \times 2$   $k$ -points, and a threshold  $10^{-6}$  eV for energy convergence. The DFT plus on-site repulsion  $U$  (DFT+ $U$ ) method<sup>76</sup> was employed at  $U_{\text{eff}} = U - J = 3, 4$ , and 5 eV to describe the electron correlation in the Mn 3d states.

For  $\text{Li}_2\text{MnSiO}_4$ ,  $\text{Na}_2\text{MnSiO}_4$ ,  $\text{AgMnSiO}_4$ ,  $\text{Li}_2\text{MnGeO}_4$ , and  $\text{Na}_2\text{MnGeO}_4$ , we considered various spin exchange paths described in Tables 6–10. To extract the values of  $N$  spin exchanges, we examined the relative energies of the  $N + 1$  ordered spin arrangements presented in Figures S1–S5 of the Supporting Information on the basis of DFT+ $U$  calculations

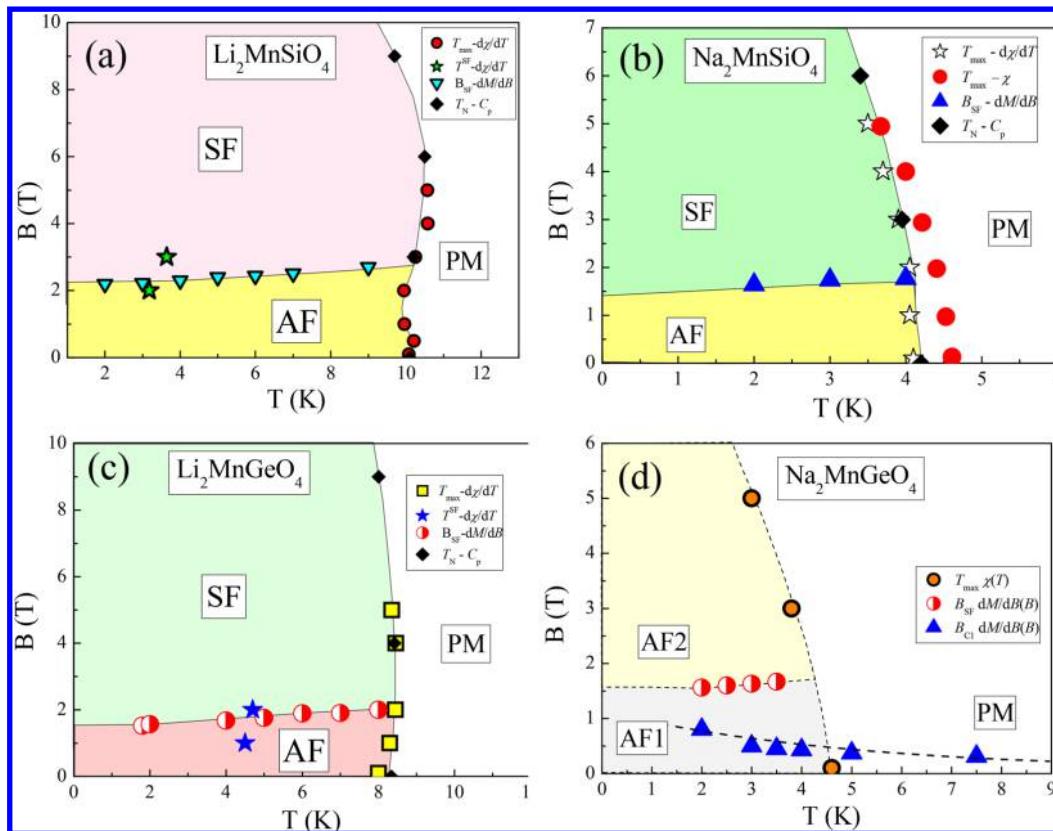


Figure 10.  $B-T$  magnetic phase diagrams for (a)  $\text{Li}_2\text{MnSiO}_4$ , (b)  $\text{Na}_2\text{MnSiO}_4$ , (c)  $\text{Li}_2\text{MnGeO}_4$ , and (d)  $\text{Na}_2\text{MnGeO}_4$ .

**Table 6. Geometrical Parameters Associated with the Spin Exchange Paths of  $\text{Li}_2\text{MnSiO}_4$  and Their Values (in  $k_{\text{B}}$  K) Obtained from the DFT+U Calculations with  $U_{\text{eff}} = 4$  eV<sup>a</sup>**

	Mn…Mn (Å)	Mn–O (Å)	O…O (Å)	value ( $k_{\text{B}}$ K)
$J_1$ (white)	4.142	2.023, 2.091	2.723	-0.22
$J_2$ (yellow)	4.423	2.023, 2.091	2.698	-0.46
$J_3$ (purple)	4.485	2.023, 2.012	2.704	-0.29
$J_4$ (cyan)	4.742	2.023, 2.012	2.734	-0.86
$J_5$ (green)	5.073 (  c)	2.091, 2.012	2.665	-3.24
$J_6$	6.305	2.012, 2.023	3.295	-0.03
$J_7$	6.336 (  a)	2.012, 2.023	3.220	-0.01
$J_8$ (orange)	6.359	2.023, 2.023	2.679	-0.49

<sup>a</sup>The colors in the first column refer to those in Figure 11.

with  $U_{\text{eff}} = 3, 4$ , and  $5$  eV. The total spin exchange energy per supercell,  $E_{\text{SE}}$ , of each ordered spin arrangement is expressed as

$$E_{\text{SE}} = (n_1 J_1 + n_2 J_2 + n_3 J_3 + \dots + n_{N+1} J_{N+1}) S^2 \quad (6)$$

where  $S$  is the total spin on  $\text{Mn}^{2+}$  ( $S = \frac{5}{2}$ ). From these expressions, we obtain  $N$  relative energies  $\Delta E_{\text{SE}}$ . The  $N$  relative energies  $\Delta E_{\text{DFT}}$  per supercell, calculated for the  $N+1$  ordered spin states by DFT+U calculations, and the values of  $n_1 - n_{N+1}$  for each ordered spin arrangement are summarized in Tables S5–S14 of the Supporting Information. The values of  $J_1 - J_{N+1}$ , obtained by mapping the energies  $\Delta E_{\text{SE}}$  onto  $\Delta E_{\text{DFT}}$ , are summarized in Tables S15–S19 of the Supporting Information. In general, the calculated spin exchange constants are very weak; only a few have values greater than 1 K.

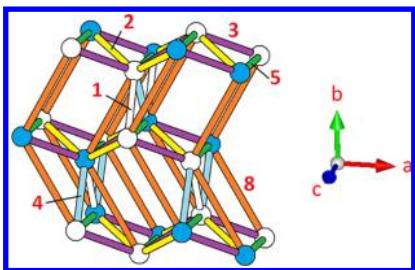
In the mean-field theory approximation,<sup>77,78</sup> the Curie–Weiss temperature,  $\theta$ , is related to the spin exchange parameters as follows:

$$\theta = \left( \frac{S(S+1)}{3k_{\text{B}}} \right) \sum_i z_{J_i} \quad (7)$$

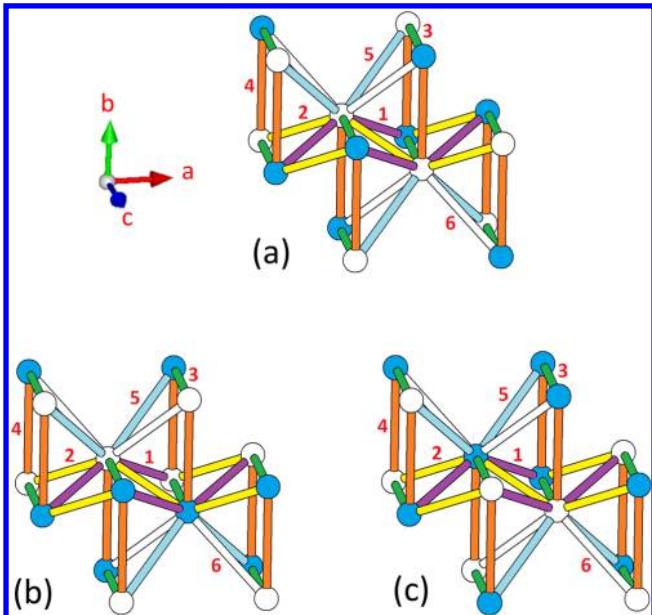
where  $z_i$  is the number of the nearest neighbors interacting along the exchange paths  $J_i$  (Table S20 of the Supporting Information), and  $S = \frac{5}{2}$  for  $\text{Mn}^{2+}$ . The calculated values  $\theta_{\text{cal}}$  obtained from the DFT+U calculations with  $U_{\text{eff}} = 4$  eV are compared with the experimental ones  $\theta_{\text{exp}}$  (Table 11 and Table S21 of the Supporting Information). The agreement between  $\theta_{\text{cal}}$  and  $\theta_{\text{exp}}$  is reasonably good, except for  $\text{Na}_2\text{MnSiO}_4$ , for which  $\theta_{\text{cal}} \approx \theta_{\text{exp}}/2$ . In what follows, we employ the spin exchanges  $J_1 - J_{N+1}$  obtained from these calculations to discuss the observed magnetic properties of  $\text{A}_2\text{MnXO}_4$ . The values of  $J_1 - J_{N+1}$  calculated by using  $U_{\text{eff}} = 4$  eV and the geometrical parameters associated with these exchange paths are summarized in Tables 6–10.

**3.8.2. Results.  $\text{Li}_2\text{MnSiO}_4$ .** The arrangements of the spin exchange paths are depicted in Figure 11, where  $J_6$  and  $J_7$  are omitted because they are very weak. The remaining spin exchanges are all AFM with  $J_5$  as the strongest one along the  $c$ -axis (hereafter along the ||c-axis), and give rise to spin-frustrated triangles<sup>59,79</sup> ( $J_5, J_4, J_1$ ), ( $J_5, J_2, J_3$ ), ( $J_8, J_2, J_1$ ), and ( $J_4, J_8, J_3$ ) (Table 6). The most stable collinear spin arrangement deduced from these spin exchanges is illustrated in Figure 11, where the four strongest AFM exchanges  $J_5, J_4, J_8$ , and  $J_2$  are coupled antiferromagnetically. Because  $J_1$  and  $J_3$  are considerably weaker than these four, the extent of spin frustration in the spin triangles is not strong, which accounts for the low spin frustration index  $f = 3.0$ .

**$\text{Na}_2\text{MnSiO}_4$ .** The weakest exchange  $J_7$  is neglected in our analysis. The exchanges  $J_1 - J_6$  are all AFM, with the largest one  $J_3$  along the ||c-axis (Figure 12). These AFM spin exchanges



**Figure 11.** Spin exchange paths of the collinear spin arrangement expected for  $\text{Li}_2\text{MnSiO}_4$ , where the shaded and unshaded spheres represent up-spin and down-spin  $\text{Mn}^{2+}$  sites, respectively. The numbers 1, 2, etc. represent the spin exchanges  $J_1, J_2$ , etc., respectively.



**Figure 12.** Spin exchange paths of the three low-energy collinear spin arrangements expected for  $\text{Na}_2\text{MnSiO}_4$ , where the shaded and unshaded spheres represent up-spin and down-spin  $\text{Mn}^{2+}$  sites, respectively. The numbers 1, 2, etc. represent the spin exchanges  $J_1, J_2$ , etc., respectively.

lead to spin-frustrated triangles ( $J_3, J_2, J_1$ ), ( $J_3, J_6, J_5$ ), ( $J_4, J_1, J_5$ ), and ( $J_4, J_2, J_6$ ) (Table 7). The three collinear spin arrangements depicted in Figure 12 are similar in energy. There is no dominant spin exchange interaction, and the extent in spin-frustrated triangle is significant triangle. This explains the high degree of spin frustration ( $f = 6.4$ ).

**Table 7. Geometrical Parameters Associated with the Spin Exchange Paths of  $\text{Na}_2\text{MnSiO}_4$  and Their Values (in  $k_{\text{B}}$  K) Obtained from the DFT+U Calculations with  $U_{\text{eff}} = 4$  eV<sup>a</sup>**

path	$\text{Mn}\cdots\text{Mn}$ (Å)	$\text{Mn}-\text{O}$ (Å)	$\text{O}\cdots\text{O}$ (Å)	value ( $k_{\text{B}}$ K)
$J_1$ (purple)	4.858	2.082, 2.079	2.652	-0.24
$J_2$ (yellow)	4.872	2.007, 2.079	2.687	-0.31
$J_3$ (green)	5.334 ( $\parallel c$ )	2.082, 2.007	2.644	-0.89
$J_4$ (orange)	5.610 ( $\parallel b$ )	2.079, 2.086	2.548	-0.38
$J_5$ (cyan)	5.662	2.082, 2.086	2.590	-0.20
$J_6$ (white)	5.674	2.086, 2.007	2.612	-0.27
$J_7$	7.029 ( $\parallel a$ )	2.007, 2.086	3.961	0.02

<sup>a</sup>The colors in the first column refer to those in Figure 12.

$\text{Ag}_2\text{MnSiO}_4$ .  $J_2$  is negligible compared with other spin exchanges.  $J_1$  as well as  $J_3-J_6$  are AFM, while  $J_7$  is FM, with  $J_3$  as the strongest exchange along the  $\parallel c$ -axis (Table 8). These spin

**Table 8. Geometrical Parameters Associated with the Spin Exchange Paths of  $\text{Ag}_2\text{MnSiO}_4$  and Their Values (in  $k_{\text{B}}$  K) Obtained from the DFT+U Calculations with  $U_{\text{eff}} = 4$  eV<sup>a</sup>**

path	$\text{Mn}\cdots\text{Mn}$ (Å)	$\text{Mn}-\text{O}$ (Å)	$\text{O}\cdots\text{O}$ (Å)	value ( $k_{\text{B}}$ K)
$J_1$ (yellow)	4.941	2.016, 2.111	2.653	-0.14
$J_2$	4.967	2.111, 1.962	2.720	-0.01
$J_3$ (green)	5.510 ( $\parallel c$ )	1.962, 2.016	2.737	-0.44
$J_4$ (white)	5.591	2.002, 2.016	2.630	-0.31
$J_5$ (cyan)	5.615	1.962, 2.002	2.767	-0.10
$J_6$ (orange)	5.648 ( $\parallel b$ )	2.111, 2.002	2.773	-0.14
$J_7$ (blue)	6.939 ( $\parallel a$ )	2.016, 2.002	3.927	0.17

<sup>a</sup>The colors in the first column refer to those in Figure 13a.

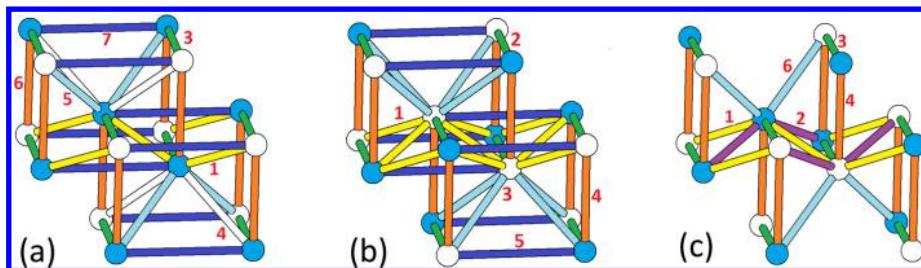
exchanges give rise to spin-frustrated triangles ( $J_3, J_4, J_5$ ) and ( $J_4, J_1, J_6$ ). In the most stable collinear spin arrangement deduced from these spin exchanges, depicted in Figure 13a,  $J_6$  and  $J_3$  have an AFM coupling, while  $J_7$  has an FM coupling. Nevertheless, all spin exchanges are very weak, so effectively many different spin arrangements should have a similar energy. This would be responsible for why the spin-frustration index is high ( $f > 5$ ).

$\text{Li}_2\text{MnGeO}_4$ . The arrangements of the spin exchanges are shown in Figure 13b.  $J_1-J_5$  are all AFM, with  $J_2$  as the strongest one along the  $\parallel c$ -axis. These AFM spin exchanges lead to spin-frustrated triangles ( $J_2, J_1, J_1$ ), ( $J_2, J_3, J_3$ ), ( $J_5, J_1, J_1$ ), ( $J_3, J_1, J_4$ ), and ( $J_5, J_3, J_3$ ) (Table 9). The most stable collinear spin arrangement deduced from these spin exchanges is illustrated in Figure 13b, in which  $J_2, J_4$ , and  $J_5$  have an AFM coupling, while the  $J_1$  and  $J_3$  paths have either an AFM or a FM coupling.  $J_2$  is much stronger than  $J_1$  and  $J_3$ ;  $J_5$  is stronger than  $J_1$ , and  $J_3$  is stronger than  $J_1$  and  $J_4$ . Thus, spin-frustration from the ( $J_2, J_1, J_1$ ), ( $J_2, J_3, J_3$ ), ( $J_5, J_1, J_1$ ), and ( $J_3, J_1, J_4$ ) is not strong. Only the spin exchanges in the ( $J_3, J_3, J_5$ ) triangle are comparable in magnitude. This explains why the frustration is not large for  $\text{Li}_2\text{MnGeO}_4$  ( $f = 3.5$ ).

$\text{Na}_2\text{MnGeO}_4$ . The arrangements of the spin exchanges are depicted in Figure 13c. The exchanges  $J_5$  and  $J_7$  are negligible compared with the rest. The strongest exchange  $J_3$  is along the  $\parallel c$ -axis. These AFM spin exchanges lead to spin-frustrated triangles ( $J_3, J_1, J_2$ ) and ( $J_6, J_2, J_4$ ) (Table 10). The most stable collinear spin arrangement deduced from these spin exchanges is depicted in Figure 13c.  $J_1$  and  $J_2$  are comparable in strength in the ( $J_3, J_1, J_2$ ), while  $J_2$  and  $J_4$  are comparable in the triangle. This may explain why the spin-frustration index is high ( $f = 6.8$ ).

**3.9. General Remarks on Magnetic Properties.** There are several points of importance to note when discussing the spin exchange interactions of  $\text{A}_2\text{MnGeO}_4$  ( $\text{A} = \text{Li, Na}$ ) and  $\text{A}_2\text{MnSiO}_4$  ( $\text{A} = \text{Li, Na, Ag}$ ):

- In all these phases, the most significant spin exchange runs along the  $\parallel c$ -axis.
- The spin frustration in all of these phases can be attributed to the presence of spin-frustrated exchange triangles.
- Compared with those of  $\text{A}_2\text{MnGeO}_4$  and  $\text{A}_2\text{MnSiO}_4$  ( $\text{A} = \text{Li, Na}$ ), all spin exchanges of  $\text{Ag}_2\text{MnSiO}_4$  are very weak. This makes many different spin arrangements have a similar energy (i.e., highly spin frustrated) so that



**Figure 13.** Spin exchange paths of the collinear spin arrangement expected for (a)  $\text{Ag}_2\text{MnSiO}_4$ , (b)  $\text{Li}_2\text{MnGeO}_4$ , and (c)  $\text{Na}_2\text{MnGeO}_4$ , where the shaded and unshaded spheres represent up-spin and down-spin  $\text{Mn}^{2+}$  sites, respectively. The coordinate axes used for presenting the  $\text{Mn}^{2+}$  ion arrangement are identical to those shown in Figure 12. The numbers 1, 2, etc. represent the spin exchanges  $J_1$ ,  $J_2$ , etc., respectively.

**Table 9. Geometrical Parameters Associated with the Spin Exchange Paths of  $\text{Li}_2\text{MnGeO}_4$  and Their Values (in  $k_{\text{B}}$  K) Obtained from the DFT+U Calculations with  $U_{\text{eff}} = 4$  eV<sup>a</sup>**

path	$\text{Mn}\cdots\text{Mn}$ (Å)	$\text{Mn}-\text{O}$ (Å)	$\text{O}\cdots\text{O}$ (Å)	value ( $k_{\text{B}}$ K)
$J_1$ (yellow)	4.518	2.052, 2.106	2.834	-0.23
$J_2$ (green)	5.054 (  c)	2.156, 2.106	2.783	-2.37
$J_3$ (cyan)	5.450	2.052, 2.052	3.148	-0.50
$J_4$ (orange)	5.476 (  b)	2.106, 2.052	3.190	-0.31
$J_5$ (blue)	6.468 (  a)	2.052, 2.052	2.896	-0.48

<sup>a</sup>The colors in the first column refer to those in Figure 13b.

**Table 10. Geometrical Parameters Associated with the Spin Exchange Paths of  $\text{Na}_2\text{MnGeO}_4$  and Their Values (in  $k_{\text{B}}$  K) Obtained from the DFT+U Calculations with  $U_{\text{eff}} = 4$  eV<sup>a</sup>**

path	$\text{Mn}\cdots\text{Mn}$ (Å)	$\text{Mn}-\text{O}$ (Å)	$\text{O}\cdots\text{O}$ (Å)	value ( $k_{\text{B}}$ K)
$J_1$ (yellow)	4.905	1.892, 2.091	2.964	-0.51
$J_2$ (purple)	4.912	2.091, 1.840	2.984	-0.39
$J_3$ (green)	5.407 (  c)	1.840, 1.892	3.160	-1.41
$J_4$ (orange)	5.730 (  b)	2.091, 2.119	2.661	-0.31
$J_5$	5.840	2.119, 1.892	3.002	-0.04
$J_6$ (cyan)	5.846	1.840, 2.119	2.852	-1.10
$J_7$	7.164 (  a)	1.892, 2.119	4.098	0.01

<sup>a</sup>The colors in the first column refer to those in Figure 13c.

**Table 11. Calculated and Experimental Curie–Weiss Temperatures**

system	calculated	experimental
$\text{Li}_2\text{MnSiO}_4$	-29.5	-30
$\text{Na}_2\text{MnSiO}_4$	-13.2	-27
$\text{Ag}_2\text{MnSiO}_4$	-5.7	-9
$\text{Li}_2\text{MnGeO}_4$	-26.9	-27
$\text{Na}_2\text{MnGeO}_4$	-21.8	-27

$\text{Ag}_2\text{MnSiO}_4$  does not undergo a long-range AFM ordering down to 2 K.

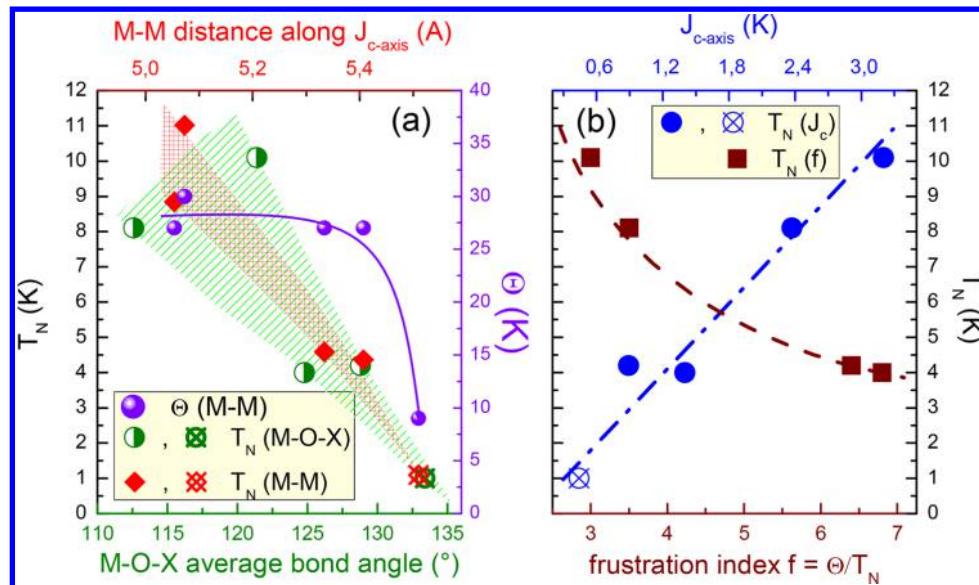
(d) If the index  $f = \Theta/T_{\text{N}} > 5$ , the system is considered to be frustrated and may not undergo a long-range AFM ordering.<sup>59</sup> Nevertheless, for the systems with  $S > 1/2$  and in the presence of different pathways for noticeable super-superexchange interactions, this naïve rule does not work, as we observed here for  $\text{Na}_2\text{MnSiO}_4$  and  $\text{Na}_2\text{MnGeO}_4$ , which order although their  $f$  indices are greater than 6 (i.e., 6.4 and 6.8, respectively).

Aiming to get general view on common trends between the magnetic properties of the compounds under study, we plotted the Néel temperature versus several structurally related geometrical parameters for dominating exchange pathways.

The results are summarized in Figure 14. In full agreement with the naïve expectations, the Néel temperature decreases with increasing both the average bond angle  $\text{M}–\text{O}–\text{X}$  and the  $\text{M}–\text{M}$  distance along the  $c$ -axis, which corresponds to dominating exchange interaction in  $\text{A}_2\text{MXO}_4$  family ( $\text{A} = \text{Li}, \text{Na}, \text{Ag}; \text{X} = \text{Si}, \text{Ge}$ ) (Figure 14a). This corroborates well the results obtained from the theoretical calculations. The strongest exchange integral  $J_{c\text{-axis}}$  correlates with the  $T_{\text{N}}$  value, and their dependence has almost linear character (Figure 14b). The detailed magnetic investigation of the  $\text{A}_2\text{MXO}_4$  highlights the crucial role of frustration. Our data show that, except for  $\text{Ag}_2\text{MnSiO}_4$ , which features only very weak magnetic interactions and must be omitted in this respect, the experimental values of the Weiss temperatures are very similar, i.e., about  $\sim 30$  K for all compounds studied (see Table 3). The experimental data hence imply that the sum of the exchange integrals, which is represented by the Weiss temperature  $\Theta$ , does not change in the series. Accordingly, as well-known for frustrated magnetic systems, the actual onset of long-range magnetic order which, in contrast to  $\Theta$  changes a lot in  $\text{A}_2\text{MnXO}_4$ , is determined by the strength of the thermal fluctuations and depends only to a minor degree on the value of the magnetic exchange. In the compounds under study, the increase of magnetic exchange along the  $c$ -axis is counterbalanced by the decrease in the perpendicular directions, but the frustration parameter changes from a rather moderate to an intermediate level. The series of compounds is a model illustrating the relationship of  $T_{\text{N}}$  and frustration without significant changes of the overall size of magnetic interaction (Figure 14b). Note that the isotropic nature of the  $\text{Mn}^{2+}$  ions assume the weak influence of magnetic anisotropy in the compounds under study. Our data hence enable studying how thermal fluctuations increase upon the increase of frustration in a rather classical  $S = 5/2$  Heisenberg system.

#### 4. CONCLUSIONS

We prepared the four new phases  $\text{A}_2\text{MnGeO}_4$  ( $\text{A} = \text{Li}, \text{Na}$ ) and  $\text{A}_2\text{MnSiO}_4$  ( $\text{A} = \text{Na}, \text{Ag}$ ), determined their crystal structures, and characterized the magnetic properties of these four phases and  $\text{Li}_2\text{MnSiO}_4$  by magnetic susceptibility, specific heat, magnetization, and ESR measurements and also by evaluating their spin exchange constants. The magnetic susceptibility and specific heat data reveal that  $\text{Ag}_2\text{MnSiO}_4$  remains paramagnetic down to 2 K, whereas phases  $\text{A}_2\text{MnXO}_4$  ( $\text{A} = \text{Li}, \text{Na}; \text{X} = \text{Si}, \text{Ge}$ ) undergo a 3D AFM ordering. The five phases  $\text{A}_2\text{MnGeO}_4$  ( $\text{A} = \text{Li}, \text{Na}$ ) and  $\text{A}_2\text{MnSiO}_4$  ( $\text{A} = \text{Li}, \text{Na}, \text{Ag}$ ) exhibit short-range AFM ordering correlations, indicating that they are low-dimensional magnetic materials. The magnetization isotherms indicate a magnetic field induced spin-reorientation (spin-flop



**Figure 14.** (a) Dependence of the Néel temperature  $T_N$  and the Weiss temperature  $\Theta$  on average bond angle M–O–X and M–M distance along the  $c$ -axis, which corresponds to dominating exchange interaction in  $A_2MXO_4$  family ( $A = Li, Na, Ag; X = Si, Ge$ ). (b) Dependence of the Néel temperature  $T_N$  on spin frustration indexes and main exchange integrals in  $A_2MXO_4$  family ( $A = Li, Na, Ag; X = Si, Ge$ ).

type) transition at  $T < T_N$  for all AFM phases. The spin-flop fields of about 2 T imply a small magnetic anisotropy gaps of  $\Delta \approx 0.23$  meV. Summarizing the thermodynamic data in magnetic fields up to 9 T allows one to construct the magnetic phase diagrams of  $A_2MnXO_4$  ( $A = Li, Na; X = Si, Ge$ ). The magnetic properties of  $A_2MnGeO_4$  ( $A = Li, Na$ ) and  $A_2MnSiO_4$  ( $A = Li, Na, Ag$ ) experimentally determined are well-accounted for by their spin exchange constants evaluated by performing energy-mapping analysis.

## ASSOCIATED CONTENT

### Supporting Information

The Supporting Information is available free of charge on the ACS Publications website at DOI: [10.1021/acs.inorgchem.7b02130](https://doi.org/10.1021/acs.inorgchem.7b02130).

Supplementary Tables S1–S21: XRD data collection and refinement details for  $A_2MnXO_4$ ; atomic coordinates and displacement parameters for  $A_2MnXO_4$ ; lattice parameters of new  $A_2MnXO_4$  in comparison with the literature data for their analogues; space groups and values of  $T_N$ ,  $\Theta$  and  $\mu_{eff}$  of  $A_2MXO_4$  ( $M = Fe, Co$ ) from magnetic susceptibility measurements and  $g$ -factor of  $Li_2FeSiO_4$  from ESR measurements; coefficients defining the total spin exchange energies of the ordered spin states per supercells of  $Li_2MnSiO_4$ ,  $Na_2MnSiO_4$ ,  $Ag_2MnSiO_4$ ,  $Li_2MnGeO_4$ , and  $Na_2MnGeO_4$ ; relative energies of the ordered spin states obtained from DFT+ $U$  calculations per supercells of  $Li_2MnSiO_4$ ,  $Na_2MnSiO_4$ ,  $Ag_2MnSiO_4$ ,  $Li_2MnGeO_4$ , and  $Na_2MnGeO_4$ ; values of the spin exchanges for  $Li_2MnSiO_4$ ,  $Na_2MnSiO_4$ ,  $Ag_2MnSiO_4$ ,  $Li_2MnGeO_4$ , and  $Na_2MnGeO_4$  determined from DFT+ $U$  calculations; coefficients  $z_1$ – $z_8$  defining the number of nearest neighbors connected by the spin exchange  $J_1$ – $J_8$ ; calculated and experimental Curie–Weiss temperatures; Supplementary Figures S1–S5: ordered spin states of  $Li_2MnSiO_4$ ,  $Na_2MnSiO_4$ ,  $Ag_2MnSiO_4$ ,  $Li_2MnGeO_4$ , and  $Na_2MnGeO_4$  ([PDF](https://doi.org/10.1021/acs.inorgchem.7b02130.supp))

## Accession Codes

CCDC 1578951–1578954 contain the supplementary crystallographic data for this paper. These data can be obtained free of charge via [www.ccdc.cam.ac.uk/data\\_request/cif](http://www.ccdc.cam.ac.uk/data_request/cif), by emailing [data\\_request@ccdc.cam.ac.uk](mailto:data_request@ccdc.cam.ac.uk), or by contacting The Cambridge Crystallographic Data Centre, 12 Union Road, Cambridge CB2 1EZ, UK; fax: +44 1223 336033.

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### Notes

The authors declare no competing financial interest.

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